

Predicting Large Scale Crack Growth In 3D Finite Element Models

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Abstract

Crack growth prediction techniques have been available for 3D models under mixed mode loading for a number of years. Previous techniques were limited to crack growth within a small region of a model. A new method is presented for automatically predicting large scale crack growth in a 3D finite element model by allowing the crack front to move through the model. Some simple examples are presented in addition to comparison of analysis results against published experimental spin test data for a cracked turbine engine compressor disc. This spin test analysis includes high levels of plasticity and crack closure effects.

Large Crack Growth - Methodology

The method of inserting a crack into a 3D model for analysis with Abaqus has been presented previously (e.g. Timbrell 1994). The method uses a crack-block approach and is summarised in Figure 1. For fatigue crack growth predictions the method has two limitations:

1. Distortion of elements limits the accuracy of the solution if the crack is at either extreme of the crack-block space i.e. if the crack is very small or large compared to the crack-block boundaries as, for example, in plot 5 of Figure 1.
2. Growth is limited to the remain within the original crack-block element space.

A further difficulty is that the original mesh must be generated with care to allow the correct size of initial defect to be defined. Although manual methods involving re-meshing allow the crack growth to extend further into the model, a fully automated method would provide a more attractive, powerful and elegant solution.

An approach that addresses the difficulties described above is one in which element boundaries are shifted and the crack-block region is transferred from one element position to the next within the mesh. A single edge notched (SEN) model is used to demonstrate the benefits of the method.

Figure 2 shows a side-on view of a symmetric SEN model in which the original regular element distribution is maintained. Cracks of increasing lengths are modelled by placing crack-blocks into different element locations. The resulting plot of stress intensity factor, K , against crack size clearly shows inaccuracies as elements become distorted within each crack-block location. There are also gaps in the stress intensity distribution where fixed element boundaries prevent analysis of particular crack lengths.

Figure 3 shows examples where the original element boundaries have been moved to help accommodate the crack-block. The stress intensity factor solution is clearly superior to that provided by the static mesh.

Large Crack Growth – Examples

The analyses of the SEN model shown in Figure 2 and Figure 3 are for fixed crack lengths – there is no automated crack growth taking place. Figure 4 shows results from automated growth analyses using data for Ti 6-4 AMS 4911G Annealed at stress ratio of zero (material data taken from AFGROW materials database, ref. 1). The plot of growth vs cycles is compared with the AFGROW solution and a stand-alone cycle-by-cycle integration program in Figure 5. The f.e. based solution gives a life of approximately 3% more cycles than the equation based solution in AFGROW and the

stand-alone software. These differences can be attributed to the fact that the f.e. solution gives slightly lower values of K than the equation used in AFGROW and that the f.e. solution uses 59 evaluations of K and an integration scheme compared to the cycle-by-cycle evaluations of K in AFGROW.

Several snapshots during growth for a more detailed SEN model are shown in Figure 6.

Compressor Disc Spin Test Benchmark

A spin test benchmark has been undertaken to demonstrate effectiveness of the techniques, coded into Zentech's Zencrack program (ref. 5), on a "real" problem. The benchmark problem is defined in Zhuang 2000. An initially quarter circular corner crack grows from a bolt hole in the disc and breaks through to a full through crack. Growth continues until the disc fails.

Spin Test - Verification of Model

The uncracked model used for the analyses consists of 216 solid 20-noded brick elements of ABAQUS element type C3D20. The geometry of the model was limited to a segment of the compressor disc between the centre lines of two adjacent bolt-holes (Figure 7). The bolt holes are 4.76mm diameter and the disc is 3mm thick at the bolt-holes (other dimensions are given in Zhuang 2000). The material is Ti-8Al-1Mo-1V titanium alloy with $E = 126000 \text{ MPa}$, $\nu = 0.33$, $\rho = 4373 \text{ kg/m}^3$ and $\sigma_y = 922 \text{ MPa}$.

The entire model is subjected to a centrifugal load corresponding to a constant amplitude load cycle for rotation of 37,000 to 8,000 rpm about the 3-axis, where the centre of rotation is defined as the centre of the whole compressor disc.

Figure 8 shows the (linear elastic) Von Mises stress distribution for the model at a rotational speed of 37,000 rpm. The background stress around the bolt-hole is found to be approximately 650 MPa,

and increases by a factor of 2 to 1300 MPa at the edge of the bolt-hole where the stress state is approximately equi-biaxial. These findings agree precisely with those given in Zhuang 2000. In an elastic-plastic analysis, the material surrounding the bolt-hole will clearly undergo plastic deformation at the peak rotation speed of 37,000 rpm.

Zhuang 2000 uses a weight function and superposition technique to generate a distribution of non-dimensional stress intensity factor, f , against crack size for a crack emanating from a hole under bi-axial loading. After completing (elastic) growth analyses with Zencrack, the results from the corner crack phase were compared to Zhuang's distribution, as shown in Figure 9. Apart from decrease in accuracy of the Zencrack results at $a/R \approx 0.75$ (due to excessive element distortion), there is good agreement with Zhuang's data.

Spin Test - Crack Growth Data

Zhuang 2000 presents crack growth data in two forms:

1. da/dN vs ΔK data at stress ratios of $R=0.0, 0.25, 0.43, 0.676$ and 0.85 .
2. da/dN vs ΔK_{eff} for the corner and through crack phases obtained from the raw data at the four stress ratios.

$$\frac{da}{dN} = C(\Delta K_{eff})^m, \quad C = 3.16 \times 10^{-10} \text{ and } m = 2.86 \text{ for corner crack growth} \quad \text{Equation 1}$$

$$C = 2.6 \times 10^{-10} \text{ and } m = 3.14 \text{ for through crack growth}$$

The effective data is obtained by using the crack closure model presented by Newman (Newman 1996) to account for plasticity induced crack closure, as defined in Equation 2.

$$\Delta K_{\text{eff}} = \left(\frac{\sigma_{\text{max}} - \sigma_o}{\sigma_{\text{max}} - \sigma_{\text{min}}} \right) \Delta K \quad \text{Equation 2}$$

where σ_o =crack opening stress, $\sigma_{\text{max}} = 650\text{MPa}$ (at 37000rpm), $\sigma_{\text{min}} = 30\text{MPa}$ (at 8000rpm).

It is not stated in Zhuang 2000 what value of crack opening stress was used to perform the conversion. Comparison of the da/dN vs ΔK and da/dN vs ΔK_{eff} given in Figures 8 and 9 of Zhuang 2000 revealed a variation in the σ_o depending on the test data stress ratio, R. Values ranged from around 200 MPa at $R = 0$, to approximately 50 MPa for $R = 0.85$.

Methods based on the crack closure model described in Newman 1996 were used to obtain an approximate value of the crack opening stress for the spin-test stress ratio of $R = 0.04615$. For an assumed geometric constraint factor of $C = 1.12$ (based on a through crack in a plate), σ_o was found to be ≈ 81.6 MPa. An alternative method of approximating the opening stress is obtained by breaking down an elastic-plastic ABAQUS analysis into a number of load steps to identify the load level at which the crack ligament is fully tensile. One load cycle was split into 200 increments to reveal, for a range of crack sizes, an average opening stress of 325 MPa.

ΔK values calculated from ZENCRACK are not ‘effective’, meaning the Paris relation must be defined in terms of ΔK rather than ΔK_{eff} . For the corner crack phase, the following values for C and m can therefore be considered:

$$\frac{da}{dN} = C(\Delta K)^m$$

$$C = 1.26413 \times 10^{-11} \quad m = 2.86 \quad \text{back-calculated from } \Delta K_{\text{eff}} \text{ with } \sigma_o = 81.6\text{MPa} \quad \text{Equation 3}$$

$$C = 2.55525 \times 10^{-12} \quad m = 2.86 \quad \text{back-calculated from } \Delta K_{\text{eff}} \text{ with } \sigma_o = 325\text{MPa} \quad \text{Equation 4}$$

$$C = 1.08227 \times 10^{-12} \quad m = 3.11527 \quad \text{straight line fit to raw data at } R=0 \quad \text{Equation 5}$$

$$C = 1.58039 \times 10^{-12} \quad m = 3.1487 \quad \text{straight line fit to raw data at } R=0.25 \quad \text{Equation 6}$$

It is noted that the “raw” test data corresponds to a load range much lower than that involved in the spin test analyses – the mean load of 173MPa compares to a mean of 325MPa for the spin test. Further, the $R=0$ and 0.25 curves are defined by only three points, the upper point falling at the lower end of the ΔK range for the spin test. The straight line approximations from the three points have been assumed to apply to higher ΔK values than provided by the test data.

Spin Test - Crack Growth – Circular Crack Phase - Elastic Finite Element Analyses

The initial quarter circular corner crack has radius 0.39mm and is shown in Figure 10. This has been analysed under the crack growth laws of Equation 3 to Equation 6. Results are compared with the reference solution of Zhuang 2000 in Figure 11. The linear elastic f.e. analyses are carried out at a load level of 37,000RPM and the stress ratio of 0.046 is applied to give a range of K .

Spin Test - Introduction of Contact

In many cases of crack growth prediction, particularly under constant amplitude loading, the effects of local crack tip plasticity are ignored and LEFM is applied. In this analysis there is a significant amount of plasticity at the bolt hole due to the high load level in the structure. If the crack is included and an elastic-plastic analysis is carried out, the plastic strain distribution will be affected by the presence of the crack. To include a representative plastic strain distribution at the bolt hole, the following procedure is used:

1. Include the crack and tie the crack faces together to effectively produce an uncracked model.
2. Load to 37000rpm to develop plastic strains around the bolt hole.
3. Release the crack face tying constraint to allow the crack to open up.

It was found that with this method the compressive stresses at the low load level caused the crack faces to close. The closure effect is greatest along the bolt-hole bore edge. This is illustrated by the contact pressure of the crack face shown in Figure 12.

Spin Test - Crack Growth – Circular Crack Phase – Elastic-Plastic Finite Element Analyses

To analyse crack growth with plastic strains introduced as described above requires definition of a suitable range of stress intensities. Due to closure effects at the low stress level the range is taken from 0 to $K_{\max}$. When the defect was analysed under these conditions crack growth was hindered by “burrowing” of the crack front, resulting in premature halting of the analysis. An example of typical burrowing crack profiles is shown in Figure 13. This burrowing can be overcome by averaging the stress intensity factors along the crack front to force uniform crack growth (as is assumed in Zhuang 2000).

Spin Test - Large Crack Growth Prediction

Discussion so far has concentrated on the corner crack phase of the growth. In order to analyse the through crack phase, the automated large growth capability in Zencrack was used. The analysis was split into two parts. The final crack position for the corner crack phase was used to estimate a starter slanting crack for the through crack phase. Results of the analyses using linear elastic and elastic-plastic finite element analyses are shown in Figure 14. Both analyses are based on the “raw” crack growth data at $R=0$. Plots of the crack models at different stages of growth for the elastic analyses are shown in Figure 15.

All analyses were run on an Windows NT PC with an AMD Athlon 750MHz processor and 512Mb of memory. The complete linear elastic growth prediction included analysis of 60 crack positions and ran in an elapsed time of 2 hours and 20 minutes.

Conclusions

A method has been demonstrated for allowing large scale crack growth prediction in 3D models. This new method allows considerable improvements on growth prediction capabilities that were previously available. The method has been demonstrated on a number of through crack geometries. Further work is required to generalise the approach to large scale growth of elliptic defects, although the movement of element boundaries does provide scope for getting more growth from existing models than would otherwise be possible.

The spin test benchmark has demonstrated the effectiveness of the method on a real problem which includes difficulties due to the high level of plasticity in the structure resulting from the high load level and geometrically induced stress concentrations. The suitability of the available crack growth data is, as may be expected, critical in accurately predicting component life. Run times for this type of prediction are well within acceptable limits.

References

1. AFGROW version 4.00.11.8 4/24/00, AFRL, WPAFB.
2. Newman Jr., J. C., "Application of a closure model to predict crack growth in three engine disc materials." *International Journal of Fracture* 80 (1996) 193-218.
3. Rooke D.P., D.J. Cartwright, Compendium of Stress Intensity Factors, H.M.S.O., London (1976).
4. Timbrell C., Claydon P.W., Cook G., "Application of ABAQUS to Analysis of 3D Cracks and Fatigue Crack Growth Prediction", ABAQUS Users' Conference, Rhode Island, U.S.A., June 1-3 1994.
5. ZENCRACK user manual, issue 6, rev 0., Zentech International Limited.
6. Zhuang, W. Z., 'Prediction of crack growth from bolt holes in a disc.' *International Journal of Fatigue* 22 (2000) 241-250.

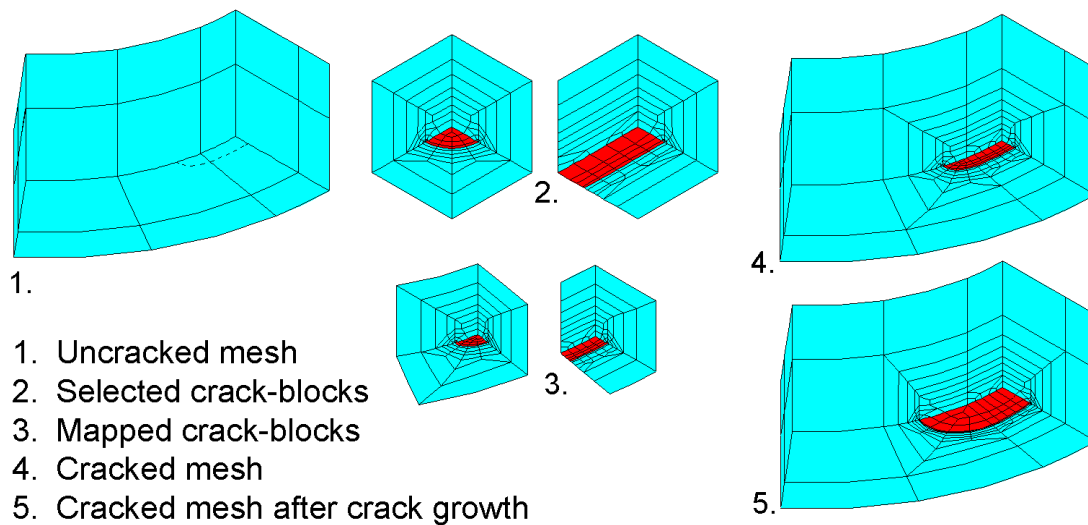


Figure 1 - Summary of crack insertion and mesh update process

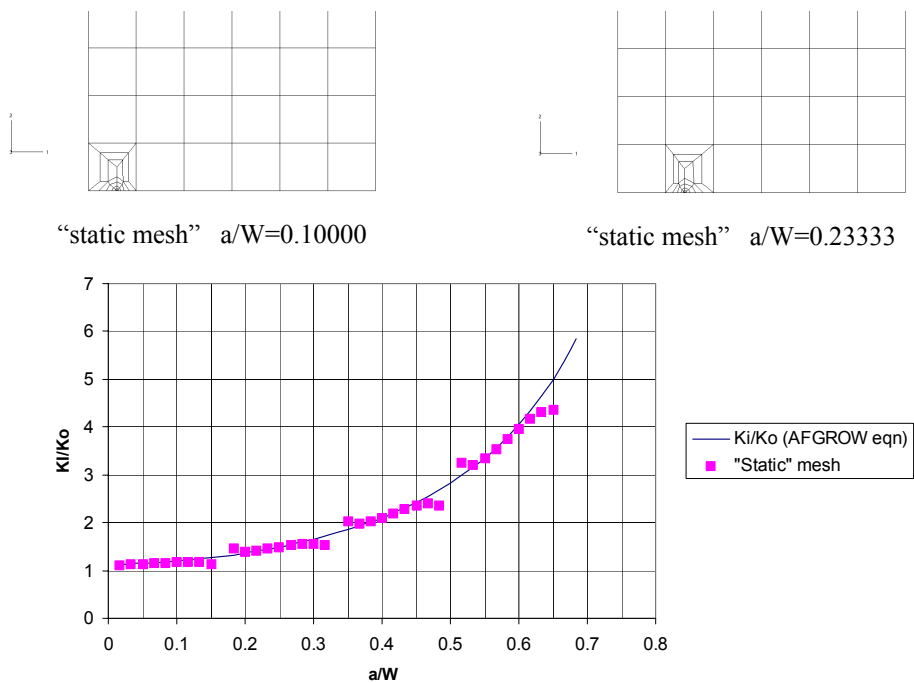


Figure 2 - “Static” mesh stress intensity solution vs a/W

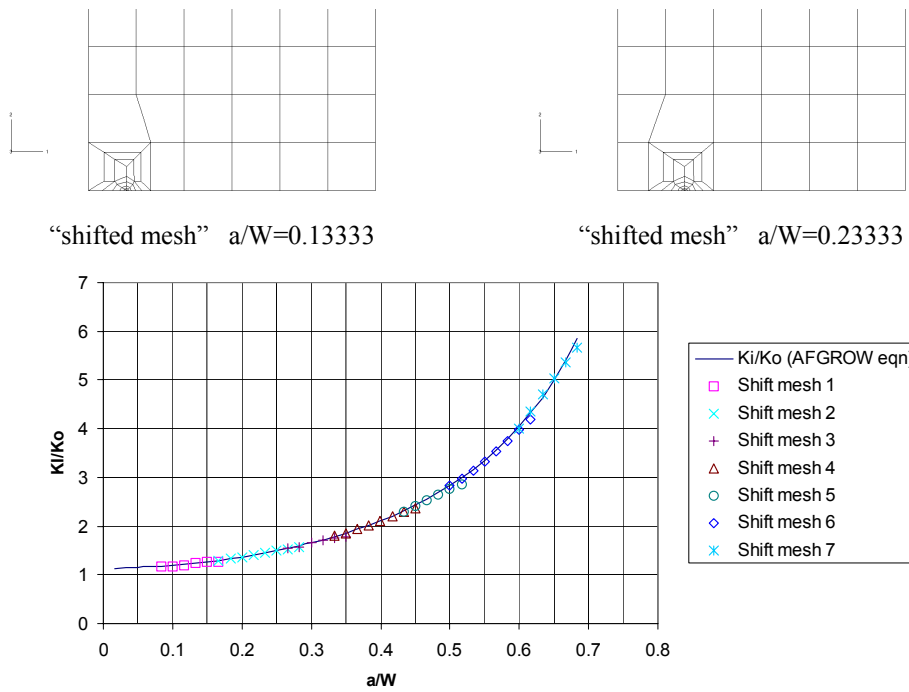


Figure 3 - "Shifted" mesh stress intensity solution vs a/W

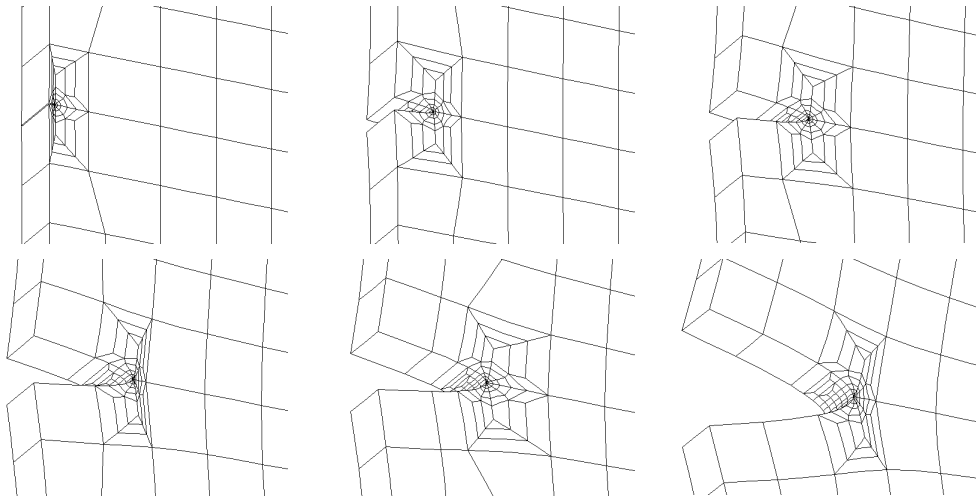


Figure 4 - Crack at six positions during growth calculation

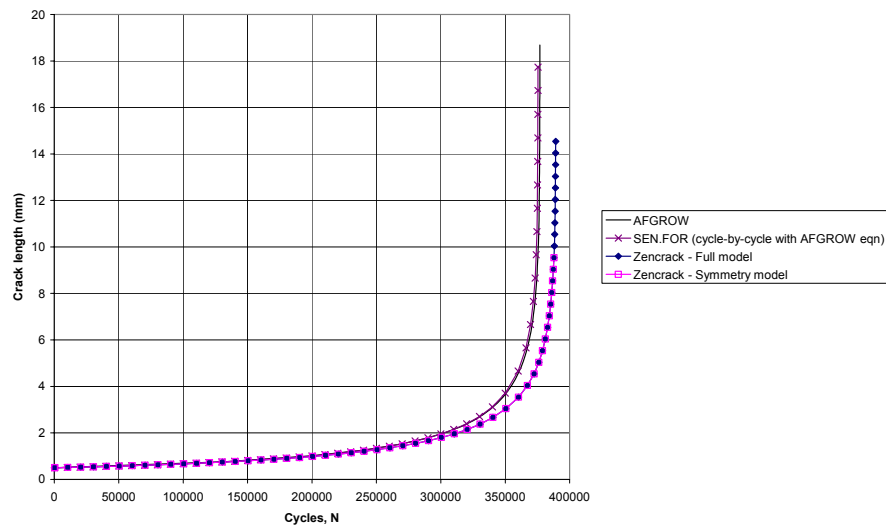


Figure 5 - Growth vs cycles for SEN model

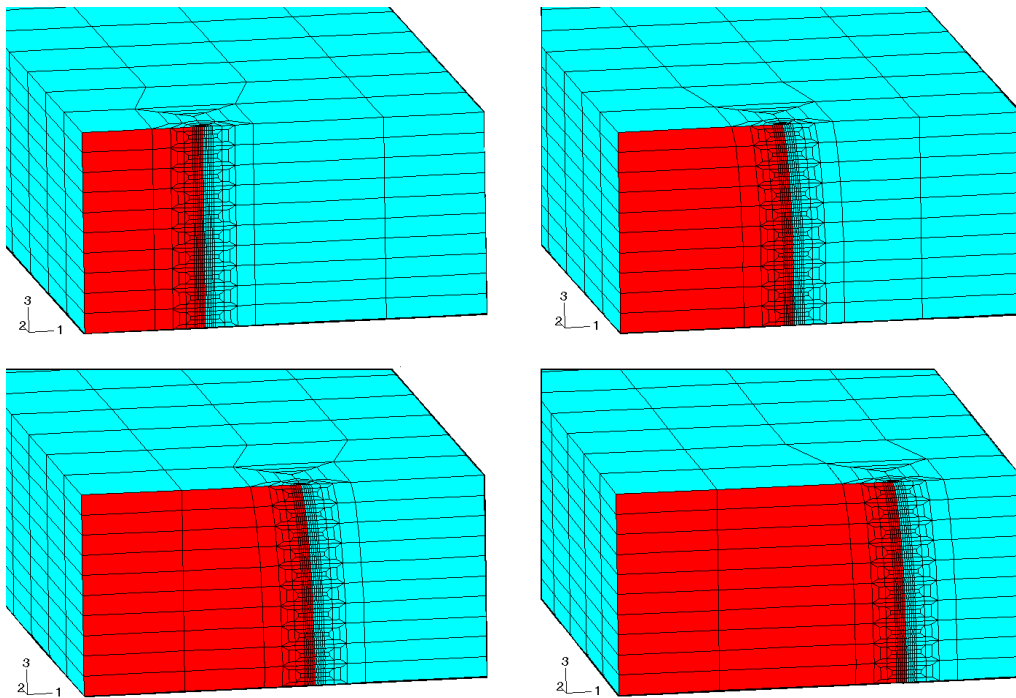


Figure 6 - Example large growth with several crack-blocks

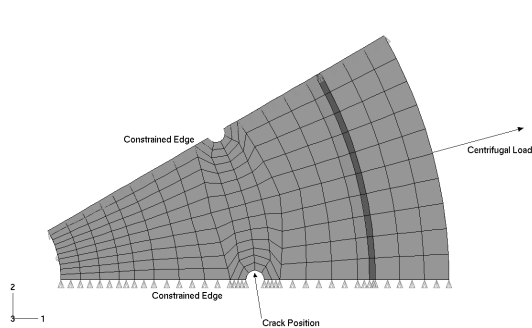


Figure 7 - Spin disc finite element model

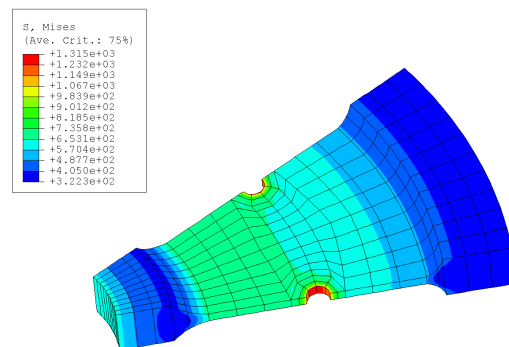


Figure 8 - Von Mises stress for un-cracked disc

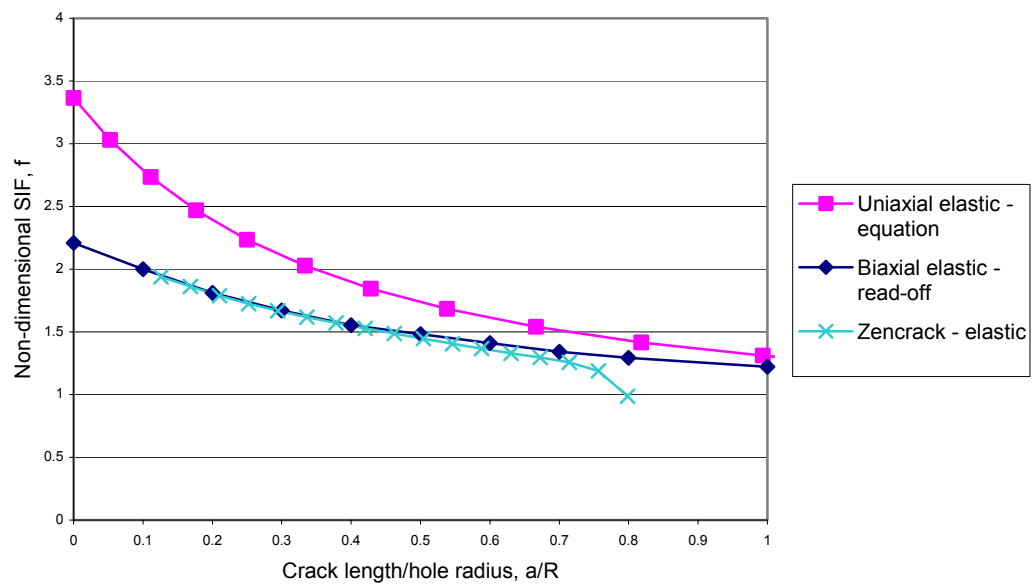


Figure 9 - Comparison of elastic non-dimensional stress intensity factor

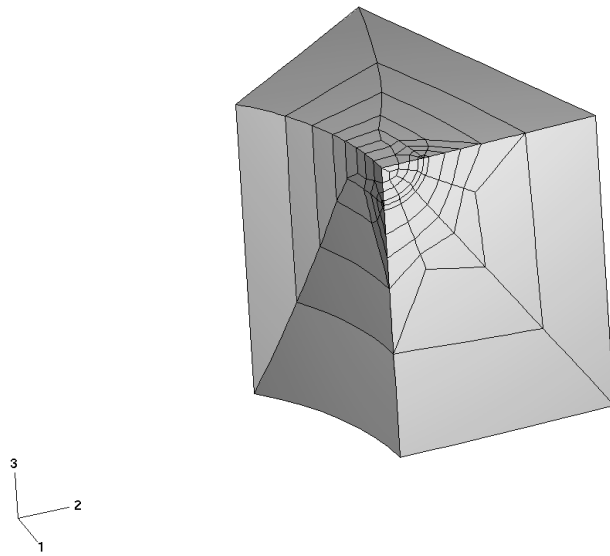


Figure 10 - Crack-block with 0.39mm quarter-circular crack

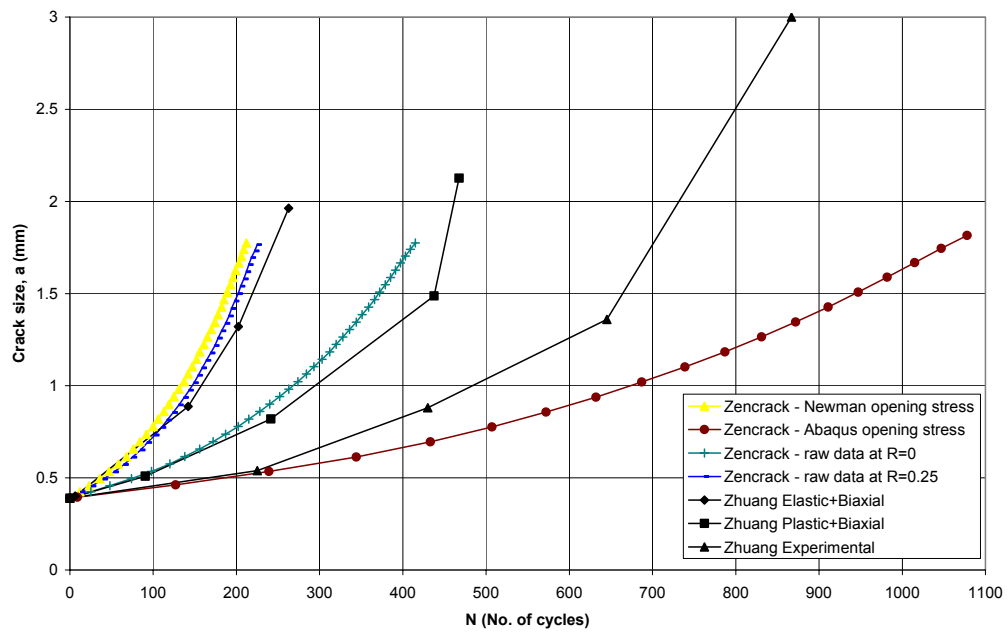


Figure 11 - Growth curves from elastic f.e. analyses

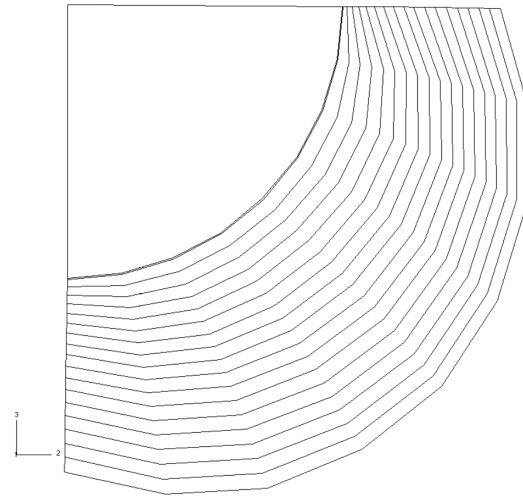
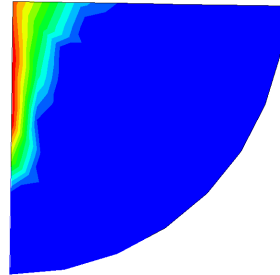
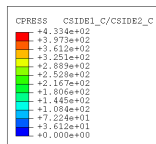


Figure 12 - Contact pressure at 8,000rpm

Figure 13 - Plastic crack growth rings

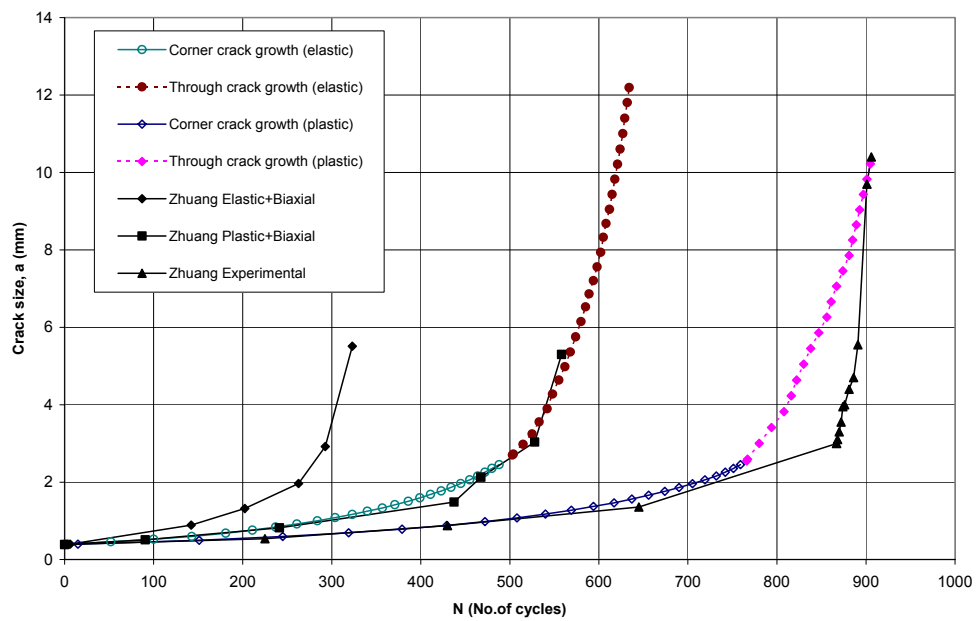
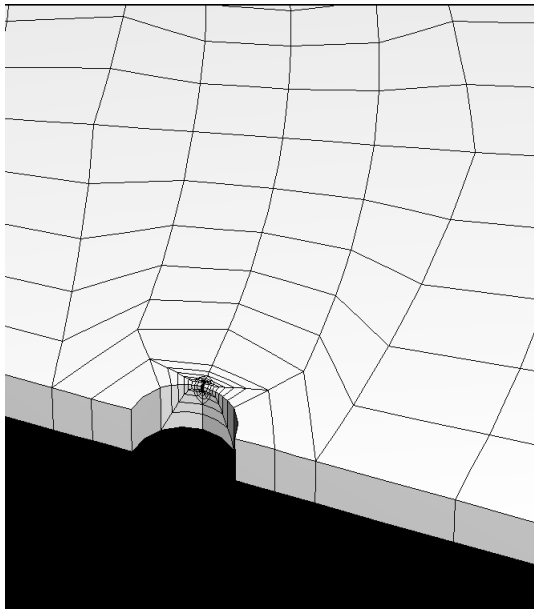
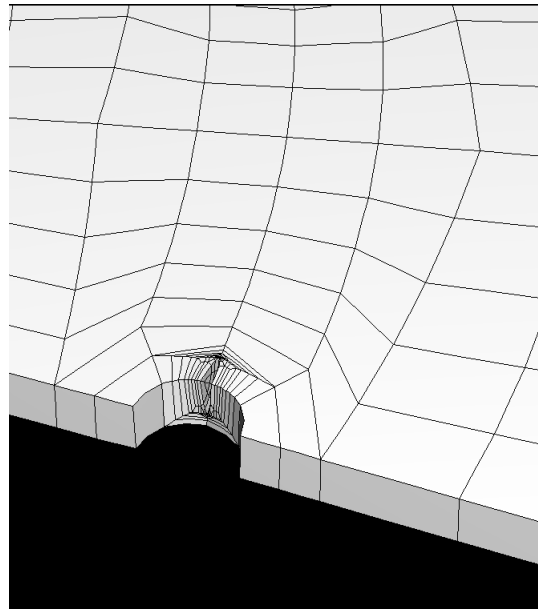


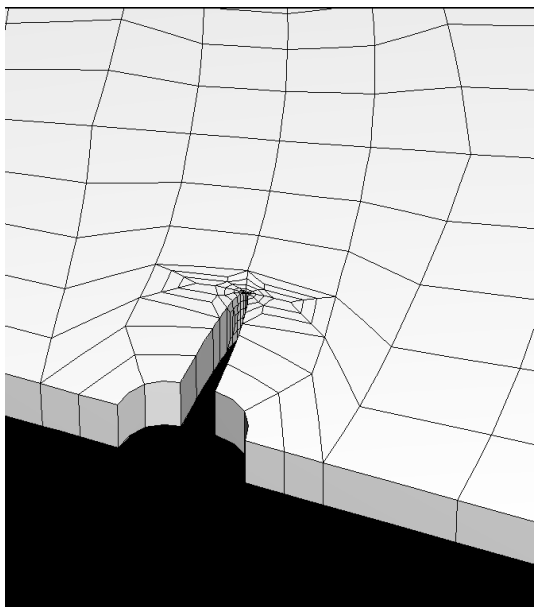
Figure 14 – Final predictions - large crack growth corner and through crack phases



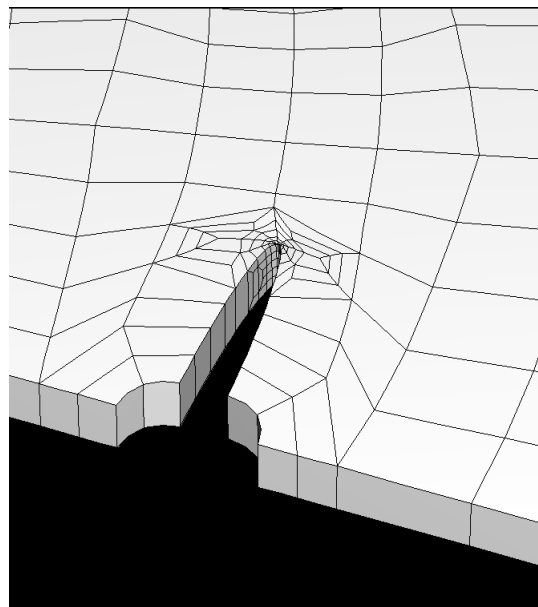
Corner crack phase – step 1



Corner crack phase – step 20



Through crack phase - step 14



Through crack phase - step 24

Figure 15 – Spin test - large crack growth mesh plots