

State Of The Art In Crack Propagation

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ABSTRACT

This paper discusses the issues involved in numerical crack growth prediction for general 3D cracks and describes the state of the art methods that are available to practising engineers. This is a wide ranging subject in which no single theoretical method is appropriate for all cases. Different approaches are adopted, for example, for crack propagation under static load, sustained load, fatigue load and impact load. The historical pedigree of the various approaches dictates the extent to which commercial software can provide practical solutions on a day-to-day basis. A brief overview is given of the relevant fracture mechanics parameters and their use in crack growth prediction under various load conditions.

The difficulties imposed by real-life problems are further compounded by the complex 3D geometries that are involved. These complexities may arise from general component shape such as turbine disk-to-blade connections or from individual geometric discontinuities such as chamfers or stiffeners. Further complications may be introduced from a variety of sources including residual stress effects, propagation along dissimilar material interfaces and propagation in non-metallic materials or metals which are non-homogenous, large grained or anisotropic.

A number of numerical approaches are discussed and the advantages and disadvantages of each are noted. Difficulties associated with growth of a general 3D crack front are considered in general and with respect to each method.

Of the various load types that may cause crack propagation, fatigue is the most advanced in terms of useable prediction capabilities. The current state of the art for fatigue crack propagation allows for growth of multiple non-planar defects through a 3D structure under general mixed mode loading. Stress ratio and load interaction effects may be included within the analysis. The most general integration schemes allow for proprietary crack growth models, including stress

ratio and temperature dependency. In addition, the effect of a static load component such as residual stress may be included within the analysis. Sustained load damage (e.g. Creep) may be combined with instantaneous damage due to rainflow counted fatigue cycles.

A number of examples are presented.

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INTRODUCTION

Why is crack propagation important?

Crack growth behaviour is a major issue in scheduling of inspection and maintenance in a variety of industries. Aerospace structures and engines are an obvious example where failure could lead to catastrophic consequences and loss of life. Numerous other examples can be cited in other engineering fields. The financial costs involved if an in-service component is found to contain a defect are a major factor in the search for numerical methods to predict 3D crack propagation. For example, the question “Should a fleet of aircraft be grounded?” has huge implications for both civil and military environments. In addition, the ability to safely reduce maintenance intervals and extend the life of in-service components can provide huge savings.

The mere presence of a crack does not condemn a component or structure to be unsafe and hence unreliable. Whether under cyclic or sustained loading it is necessary to know how long an initial crack of certain size would take to grow to a critical size at which the component or structure would become unsafe and fail. Also by knowing how a crack evolves and its rate of propagation, one should be able to estimate the residual service life of a component under normal service loading conditions. Crack propagation enables one to predict the period of sub-critical crack growth and hence the service life of a component.

Other situations where crack propagation is required include:

- Studying the effects of surface treatment such as shot peening, laser shock peening etc., to enhance the service life of a component
- Studying the effects of surface treatment such as chromium plating which can degrade the service life of a component
- Studying the effectiveness of crack repair systems, remedial work, modifications or design changes
- Establishing inspection and maintenance regimes

A dramatic example of the effect of surface treatment on crack growth is shown in the schematic of Figure 1 which is based on experimental data presented in Ref. 1. In this test an initial defect is grown a small amount under fatigue loading before surface treatment is applied. After the surface treatment the growth is retarded by the treatment-induced residual stress to such an extent that the applied load must be increased to achieve further growth. It is clearly demonstrated that the shape development is significantly affected by the presence of the residual stress.

Until recently mode I and mixed mode crack growth behaviour was generally evaluated using experimental tests similar to the one described in Ref. 1. However, using the design philosophy based on reliability and “safe life” criteria, it becomes necessary to test a large number of materials and structural

components in a very short period. This makes the use of experimental methods rather impractical and implies the benefit of using numerical computational methods to evaluate 3D mixed mode crack propagation from simple mode I tests.

Overview of requirements for crack propagation analysis

Any crack growth analysis must be based on the calculation and use of fracture mechanics parameters. Such parameters encapsulate and describe the local effect of the crack on a component. The satisfactory calculation of fracture mechanics parameters is the most fundamental requirement of crack propagation analysis.

Having evaluated fracture parameters for a particular crack in a component under a given load, the next task is to convert that information into crack growth. This requires knowledge of the load history and appropriate material crack growth data. In most cases this will be fatigue based cyclic loading although time dependent (sustained load) growth is also possible. Growth is calculated over a number of load cycles or an elapsed time respectively. The method used to determine the crack growth depends upon the numerical technique implemented for the analysis. For a closed form method in which fracture parameters can be quickly and cheaply re-calculated, cycle-by-cycle integration is possible. But in general this is not appropriate and a robust numerical integration scheme is required.

The approach taken for integration of crack growth must be able to accommodate the evolution of an arbitrary crack shape that may be non-planar. This shape evolution presents topologically difficult problems for the numerical methods that are most often used to calculate fracture parameters.

In addition, the analysis should be able to take account of effects such as residual stress and load interaction.

The requirements for crack propagation are therefore relatively easily stated. However, the solution for a general 3D crack in a component is rather more difficult to achieve.

FRACTURE MECHANICS PARAMETERS & RELATED ISSUES

Introduction

The primary fracture mechanics parameters that may be of interest for crack propagation are:

- Stress intensity factors, K_I , K_{II} , K_{III}
- Energy release rate, G

- J-integral, J

The stress intensity factor approach was developed by Irwin in the 1950s following on from the elastic strain energy approach to brittle fracture developed by Griffith from the 1920s. Irwin's work led to the foundations for the concept of linear elastic fracture mechanics (LEFM) which is still fundamental in most crack propagation analyses.

For linear elastic analysis the concepts of energy release rate and stress intensity factors are closely linked. The stress intensity factors describe the magnitude of the elastic stress field at a crack front. The general form of the stress intensity factor is:

$$K = f(\text{load}, \text{crack length}, \text{geometry}) \quad \text{Equation 1}$$

For mode I behaviour it can be shown that:

$$K = \left(\frac{EG}{1 - (\alpha\nu)^2} \right)^{1/2} \quad \text{Equation 2}$$

where E is Young's modulus, ν is the Poisson ratio and α is a value ranging from 0 for plane stress to 1 for plane strain. In a more general form it is possible to write:

$$G = \frac{B}{E} (K_I^2 + K_{II}^2) + \left(\frac{1 + \nu}{E} \right) K_{III}^2 \quad \text{Equation 3}$$

where $B=1$ for plane stress and $1-\nu^2$ for plane strain.

Another important relationship for stress intensity factors in linear elastic analysis is based on the Westergaard equations that link the stress intensity factors to the displacement field around the crack tip giving:

$$K_I = \frac{EV_i}{4B} \sqrt{\frac{2\pi}{r}}, \quad K_{II} = \frac{EV_{ii}}{4B} \sqrt{\frac{2\pi}{r}}, \quad K_{III} = \frac{E}{2(1 + \nu)} V_{iii} \sqrt{\frac{\pi}{2r}} \quad \text{Equation 4}$$

where B is defined as above and V_i , V_{ii} and V_{iii} are the relative opening displacements at a radius r from the crack front for an orthogonal system aligned with the mode I, II and III directions.

Calculation of the magnitudes of energy release rate and / or stress intensity factors do not provide directional information regarding crack growth. A number of criteria have been developed to specify the direction. They include maximum energy release rate, maximum tangential stress and the normal to the maximum principal stress. In the context of numerical calculations of energy release rate and stress intensity factor, the two most useful criteria are maximum energy release rate and a direction based on stress intensity factors e.g.:

$$\theta = \tan^{-1} \left(\frac{K_{II}}{K_I} \right)$$

Equation 5

The J-integral concept was first described by Rice in the late 1960s. It is an energy based concept in which the J-integral, J, can be considered a non-linear elastic equivalent of the energy release rate, G. By definition G and J are the same for elastic behaviour. The calculation of J in cases for which plasticity is present (EPFM) was originally developed for failure assessment. However, the equivalence of G and J for linear elastic analysis means that any numerical approach that allows calculation of J is immediately of interest in crack growth analysis that uses LEFM.

Numerical evaluation of fracture parameters

For simple geometries there may be closed form solutions available for mode I stress intensity factors and in some cases for mode II and mode III. Otherwise a boundary element or finite element analysis is required. The most common approach to evaluating the stress intensities is the use of the expressions in Equation 4 (COD method).

Evaluation of the J-integral is available in some finite element packages (e.g. Ref. 2 and Ref. 3) and is generally accepted as providing a more accurate solution than the use of displacements. The detail of the methods for calculating J-integral values is well discussed in the literature and is beyond the scope of this paper.

Complications

In reality there are a number of factors that affect crack growth and sometimes the calculation of linear elastic fracture parameters and their direct use with crack growth data is not sufficient.

For example, in a variable amplitude load history it is well known that an overload can retard the growth during subsequent load cycles. This is believed to be due to a reduction in crack growth rate as the crack grows through the extended crack tip plastic zone caused by the overload. The approaches currently available to account for these effects are empirical models which have the effect of reducing the growth rate. The Wheeler retardation model, for example, applies a factor onto the calculated crack growth rate and the Willenborg retardation model calculates an effective stress ratio. This results in a lower growth rate by virtue of the way that crack growth data behaves as a function of stress ratio.

A further difficulty is introduced by the requirement to use a state of stress assumption in equations such as Equation 2 and Equation 4. In a 3D analysis this problem can be alleviated by calculation and use of energy release rate as the fracture parameter rather than stress intensity factors. In a finite element analysis the state of stress is then embodied within the evaluation of the energy

release rate. For this type of approach the crack growth data must be converted to an energy release rate form rather than a stress intensity form. The choice of the parameter α in this conversion depends on the test specimen design (e.g. plane stress for thin sheet or plain strain for compact tension test specimens). The conversion of material data from K to G can be done programmatically rather than requiring manual conversion. Then the state of stress is no longer required. It is further noted there is a tendency for the state of stress to move towards plane stress at failure.

Importance of the accuracy of fracture parameters

In any growth analysis the accuracy of the basic fracture parameters is important in providing an accurate life prediction. Typical crack growth data involves stress intensity ranges raised to a power of around 3. So any inaccuracy in the value of stress intensity factor is magnified in the life calculation. This is demonstrated by the results in Figure 2 for a sample analysis using AFGROW (Ref. 4). For this analysis the “correct” solution obtained by cycle-by-cycle integration for an applied load level of 100 is 50000 cycles. If the analysis is repeated with load levels of 101 and 99 the life predictions are 48248 and 51841 cycles respectively. So change in load level and hence the basic stress intensity solution of $\pm 1\%$ gives change in life of -3.5% and $+3.7\%$. The discrepancies may be even more dramatic for initial cracks loaded near the fatigue threshold limit.

NUMERICAL ISSUES IN EVALUATING CRACK PROPAGATION

Choice of method to evaluate fracture parameters

The numerical issues involved in crack propagation range from the basic solution method (which essentially provides fracture parameter values) to the crack growth integration technique.

There are three main numerical approaches available to provide fracture parameters:

- 1) closed form solution
- 2) boundary element method
- 3) finite element method

In the closed form approach a known solution provides fracture parameters as a function of geometry and crack size. The known solutions may be derived from analytical expressions or, more usually, interpolated from results obtained using the boundary element or finite element methods. If it can be used, this approach provides a potentially fast method to determine crack propagation. A number of software packages are available that use this method e.g. AFGROW (Ref. 4) and NASGRO (Ref. 5). However, this approach has a number of severe limitations in terms of providing a general capability:

- The number of cases for which solutions are available is limited.
- There may be no solution that is appropriate for a given geometry and load scenario.
- Solutions are predominantly for planar cracks.
- Starter crack may be straight or elliptic but there is no account of crack shape development (other than, for example, the ellipticity may change or a straight crack may become inclined).
- Load redistribution is not accounted for as the crack grows.
- Non-linear effects cannot be taken into account.

The boundary element method is a method for solving partial differential equations using an integral equation approach. In a linear analysis it requires that only the boundary of the structure or component be discretised. This is considered to be the main advantage of the boundary element approach over the more widely used finite element method. Beasy (Ref. 6) is the best known commercial boundary element code with crack propagation capabilities. It provides a linear solution with J-integral capabilities in 2D analysis. In 3D the linear solution uses displacements to generate stress intensity factors. Nasgro also has a boundary element module.

The finite element method is the most popular approach for tackling fracture mechanics problems. This requires discretisation of the complete structure. The method can be applied to linear and non-linear problems. Many commercial packages are available for use in fracture mechanics applications. Some, such as Abaqus (Ref. 2) and MSC.Marc (Ref. 3) allow calculation of fracture mechanics parameters such as the J-integral. None have an in-built crack propagation capability. The main advantage of the leading finite element codes is the flexibility they provide in terms of non-linearity and overall analysis capabilities. The meshing issues are more difficult than for the boundary element approach but non-linear analysis is possible without any additional meshing effort.

For finite element applications, add-on software must be used for crack propagation. Zencrack (Ref. 7) is available to provide a 3D crack propagation capability for Abaqus, MSC.Marc and Ansys (Ref. 8). This software can be thought of as a fracture mechanics tool that uses the finite element method to generate fracture parameter solutions. Zencrack is capable of using J-integral or displacement based results for calculating fracture parameters.

A similar approach is used in the Franc2D and Franc3D (Ref. 9) programs although these are more usually associated with a boundary element solution. These programs use displacements from a linear boundary element or finite element solution to calculate stress intensity factors and crack growth. They have no J-integral capability.

Crack growth integration scheme

Once a method is selected to provide fracture parameters, the next step is the crack growth calculation. The discussion below relates to fatigue crack growth but similar comments apply to sustained load crack growth.

For a given material the crack growth rate, da/dN , is a complex function of many variables including stress intensity range ($\Delta K = K_{\max} - K_{\min}$), stress ratio ($R = K_{\min}/K_{\max}$), temperature and frequency of the applied cyclic load. In the most general form it is simply written as:

$$\frac{da}{dN} = f(\Delta K, R, T, f, \dots) \quad \text{Equation 6}$$

A number of explicit equation forms exist to cater for simplified situations e.g. the Paris (Equation 7) and Walker (Equation 8) equations:

$$\frac{da}{dN} = C(\Delta K)^n \quad \text{Equation 7}$$

$$\frac{da}{dN} = C_o (\Delta K_o)^n, \quad \Delta K = \Delta K_o (1 - R)^{(1-m)}$$

giving:

$$\frac{da}{dN} = C_o [\Delta K (1 - R)^{(m-1)}]^n \quad \text{Equation 8}$$

where subscript o refers to values at $R=0$

For the most general approach, the evaluation of da/dN must allow input of as many of the contributing factors as possible. It may therefore be necessary to evaluate temperature at the crack front from the fracture analysis and use that as an input to the da/dN evaluation. This incorporation of temperature effects into the integration provides a thermo-mechanical fatigue crack growth capability. In Zencrack, for example, this can be achieved using tabular da/dN vs ΔK data that is a function of stress ratio and temperature. Alternatively a user subroutine option allows complete flexibility by allowing the user to define da/dN using proprietary data based on input of ΔK , stress ratio and temperature.

Whichever approach is used to evaluate crack growth rate, the underlying methodology can be considered as a black box that simply returns a value of da/dN according to the general form Equation 6.

For the closed form method of calculating fracture parameters a cycle-by-cycle integration is possible. In this approach the value of da/dN is calculated and the crack advanced by da over 1 cycle. The fracture parameter, usually K_I for this method, is re-evaluated and a new da/dN calculated. This method is easy to

code for closed form methods and has the benefit that effects such as spectrum loading and empirical retardation models are fairly straightforward to implement. However, for the boundary element and finite element approaches, it is not feasible to re-evaluate fracture parameters every cycle. Instead an accurate integration scheme is required to allow the crack front to be advanced some finite amount before re-evaluating the parameters. It must also be remembered that a general 3D crack front has multiple points along the front, and the value of dN must be consistent although of course in general the da values will vary both in magnitude and direction.

A simple approach to this finite advance is to assume some maximum da and advance the front according to the relative stress intensity factors along the crack front. If the maximum mode I stress intensity factor is $K_{i_{\max}}$, then the maximum value of da is applied at this point, $da_{\max}$. The da for other points is evaluated by assuming consistent dN and a Paris type relationship:

$$dN = \frac{da_i}{C(\Delta K_i)^n} = \frac{da_{\max}}{C(\Delta K_{\max})^n} \quad \text{Equation 9}$$

giving:

$$da_i = da_{\max} \left(\frac{\Delta K_i}{\Delta K_{\max}} \right)^n \quad \text{Equation 10}$$

This approach is used in Franc3D in which the power in Equation 10 can be specified independently of the crack growth data (Ref. 10). The $da_{\max}$ can be fixed through the analysis, allowed to vary linearly, or be specified by the user. In the Franc3D implementation the crack growth is calculated using this method in order to determine stress intensity factor histories for crack front points. These histories are then smoothed and used in what amounts to a closed form type of approach to evaluate the life. Within the life calculation, more complex crack growth models may be selected than were assumed in advancing the crack. But unless the growth data in the life calculation is a Paris law of the type used in advancing the crack:

- Assessment of different nodes may produce different values of life.
- The predicted crack shape development may be incorrect since it is not based on the actual crack growth data and load history.

In addition, it is not clear to the authors how threshold effects are treated within the Franc3D implementation. By de-coupling the crack advancement from the crack growth data it appears possible that a point that should be below threshold may in fact have some growth.

In order to correctly evaluate the crack growth, including threshold effects, an alternative approach is required in which the cycle count is maintained as the crack grows and all crack advancement is based on the actual crack growth data. This ensures a consistent dN calculation and correct crack shape development.

A method has been implemented in Zencrack that uses a forward predictor approach to integrate over the current integration interval based on knowledge of previous discrete crack positions. This implementation is based on the use of energy release rate, G , rather than stress intensity factor.

For given load the rate of change of energy release rate with crack size, dG/da , is assumed to be constant as the crack grows. A known rate of change calculated from analysis of previous crack positions is assumed to occur over the next integration step to a new crack position. This assumption is based on simple fracture mechanics theory for a crack in a plate:

$$K = f(\text{geometry}) \sigma \sqrt{\pi a} \quad \text{Equation 11}$$

Neglecting effects of the geometry function gives an approximation that:

$$K^2 \propto a \quad \text{Equation 12}$$

For the case of mode I loading the relationship between stress intensity factor K and energy release rate G is:

$$K = \left(\frac{EG}{1 - (\alpha\nu)^2} \right)^{1/2} \quad \text{Equation 13}$$

and therefore:

$$G \propto a \quad \text{Equation 14}$$

In the Zencrack integration scheme this is written as:

$$G_{final} = G_{initial} + \left(\frac{dG}{da} \right) da \quad \text{Equation 15}$$

Clearly the choice of the step size, da , is important in this process. But comparison of the predicted value of G and the actual value of G after the advanced crack has been analysed allows calculation of an error term that is used to control the step size.

After a finite element analysis is completed Zencrack has an “accurate” value for G at each crack front node. This is denoted by G_{FEi} for the i^{th} finite element analysis. During integration Equation 15 is used to obtain the updated crack front position and at the end of the integration the growth magnitude is da_{FE} and the expected value of G from the next finite element analysis is:

$$G_{ESTIMATEi+1} = G_{FEi} + \left(\frac{dG}{da} \right) da_{FE} \quad \text{Equation 16}$$

When the analysis of the updated crack front is completed, an “accurate” value for G is available for the $i+1^{\text{th}}$ finite element analysis, $G_{\text{FE}i+1}$. As Equation 16 is just an approximation the difference between $G_{\text{FE}i+1}$ and $G_{\text{ESTIMATE}i+1}$ is an indicator of how accurate the integration was for the last step. This error term is used to control the maximum da for the next integration step. If the error is below a specified tolerance, the step size may increase. Otherwise the step size is reduced. In this way the analysis is controllable based on a quantifiable error term rather than an arbitrary selection of da . There are of course, additional details in the way that this scheme is implemented, but this description provides the basic concept.

Numerical integration of the crack growth data incorporating Equation 15 is based on an energy form of the crack growth law:

$$\frac{da}{dN} = g(\Delta\sqrt{G}, R, T, f) \quad \text{Equation 17}$$

This allows a general integral to be written:

$$\int_i^f dN = \int_i^f \frac{1}{g(\Delta\sqrt{G}, R, T, f)} da \quad \text{Equation 18}$$

Due to the fact that there are multiple node positions on the crack front, this integral must be evaluated in a 2-pass process. In the first pass, each node is integrated for the specified value of da for the step and dN is evaluated. In general this produces different dN values for each node. The minimum dN is taken to be the critical value and a second integration pass is carried out in which the da is calculated at every node for this critical dN . This provides a consistent dN calculation and in general different da for each node.

Complications

In addition to the basic numerical integration described above for fatigue crack propagation, a number of other issues affect the way that crack growth is evaluated. These include:

- effect of static load e.g. residual stress
- load interaction effects
- time dependent crack growth

Static load

In many cases the effect of a static load needs to be combined with the cyclic external load. This is most often the case due to the significant effect of residual stress on crack propagation. It is well known, for example, that surface treatment to introduce compressive residual stress near the surface can have beneficial effects in terms of component life. It is also the case that such treatment can significantly affect the crack shape development, as shown, for

example, by Prevey et al (Figure 1). The ability to include static load or residual stress effects is therefore an important feature of the crack growth integration scheme.

The consequence of the static load is to modify the instantaneous stress ratio. For a cyclic load that gives a K range between the $K_{cyclic\ max}$ and $K_{cyclic\ min}$, at a node for which the static load gives K_{static} , then:

$$R_{effective} = \frac{K_{cyclic\ min} + K_{static}}{K_{cyclic\ max} + K_{static}} \quad \text{Equation 19}$$

This is managed in Zencrack by the implementation of a load system methodology and the handling of multiple sets of fracture parameters. The analysis can be conducted in one of two ways:

Method 1 for static load

Method 1 is an LEFM approach and requires one linear FEA with two load steps. The first load step is with static load only (e.g. residual stress applied to the crack faces through a user subroutine such as DLOAD in Abaqus). The second load step is with the maximum value of the cyclic external load only. The external load results can be scaled and combined with the static load result to give a solution at any desired external load level. This method can be used with constant amplitude or spectrum external loading.

Method 2 for static load

Method 2 allows crack face contact to be incorporated in an analysis with combined static and cyclic loading. For crack growth prediction the method is limited to constant amplitude external loading in which results are generated at the minimum and maximum external loads in the cycle.

In Method 2 the first load step is with static load and minimum external load applied together. This provides a solution for the point of minimum external load in the load cycle. The second load step is with static load and maximum external load applied together and provides a solution for the point of maximum external load in the load cycle. Hence the stress intensity range and effective stress ratio can be calculated. It is noted that if the second load step is broken into a number of distinct points, the full curve between minimum and maximum external load can be generated. Indeed, if contact is introduced between the crack faces a non-linear response due to changing partial crack closure can be evaluated.

Combination equations for static and cyclic load

The G_{max} , G_{min} and R for method 1 combination are calculated as shown below. A similar approach is used for method 2 combination.

$$G_{\max new} = \left\{ G_{\max static}^{0.5} + G_{\max cyclic}^{0.5} \right\}^2 \quad \text{Equation 20}$$

$$G_{\min new} = \left\{ G_{\max static}^{0.5} + G_{\min cyclic}^{0.5} \right\}^2 \quad \text{Equation 21}$$

$$R_{new} = \left\{ \frac{G_{\min new}}{G_{\max new}} \right\}^{0.5} \quad \text{Equation 22}$$

The combined growth direction is taken as a weighted ratio of the growth direction for the base and associated systems:

$$(x_{new})_i = \frac{G_{\max static}^{0.5} (x_{static})_i + G_{\max cyclic}^{0.5} (x_{cyclic})_i}{G_{\max new}^{0.5}}, \quad i = 1, 3 \quad \text{Equation 23}$$

where x_i for $i=1,3$ represents the growth direction

Load spectrum

A load spectrum consisting of blocks of different numbers of cycles at different stress ratios must be possible in a growth analysis. Each individual block is a constant amplitude block. The integration must proceed to the end of the block before appropriate scaling factors are applied to allow integration of the next block. It is not uncommon for “real” spectra to contain several hundred thousand load blocks.

Load interaction

The effect of empirical load interaction models must be incorporated at a number of locations within the integration scheme. The details depend upon the particular retardation model with some, such as the crack closure model, presenting more severe challenges than others such as the Wheeler model. Once retardation is introduced it is necessary to incorporate an inner integration loop for the phase during which retardation takes place. In this way the end of retardation can be correctly flagged. For a 3D analysis with multiple points on the crack front, the retardation parameters must be maintained at each point.

Time dependent crack growth

Integration of time dependent crack growth requires a slightly different approach to integration of cyclic loading, the details of which are beyond the scope of this paper. The combination of time based and fatigue loading is also possible. This requires a basic time integration with the damage caused by each fatigue cycle added instantaneously at the time that the cycle is deemed to occur.

TOPOLOGY ISSUES

The analysis of general 3D crack propagation must surmount considerable difficulties in terms of topological issues. The only practical approaches are the boundary element and finite element methods. Difficulties arise for a number of reasons:

- component geometries are often complex and time consuming to model in their uncracked forms
- defects often occur at geometrically difficult locations e.g. corners, welds, chamfers
- initial cracks of the correct size and shape must be inserted into the component at the correct location
- cracks may develop in a non-planar fashion depending upon the loading

In the boundary element approach for linear analysis the difficulty of discretising the internal structure is removed. This means that only the current crack front and the previous crack path must be incorporated in the discretisation. In terms of the re-meshing required as the crack advances, this is a far simpler task than the full re-meshing required for a finite element solution. Both Beasy and Franc3D's boundary element program provide capabilities for general growth in a 3D body subject to some of the limitations mentioned earlier.

In the finite element approach the entire volume must be discretised. It is important to note the requirements for accurately modelling the singularity at the crack front. The elements of choice for such modelling are 20 noded isoparametric brick elements. The crack front is typically modelled with "rings" of elements around the front. The innermost ring contains "collapsed" elements to represent the singularity in the stress and strain field at the crack front (Ref. 11) as shown in Figure 3. Control of the nodes along the crack front and the radial nodes closest to the crack front is required to generate a singularity best suited to LEFM or EPFM (Figure 4). Control of the singularity and biasing of the rings is available in ZENCRACK to give the user flexibility in the crack modelling process. The use of hexahedral elements in this way allows evaluation of the J-integral in Abaqus and MSC.Marc and in general provides a better solution than a tetrahedral mesh using nodal displacements to calculate stress intensity factors (Note: The J-integral is not available for tetrahedral elements).

However, automated mesh generators for 3D volumes are not currently available to mesh an irregular volume with hexahedral elements. Hence an alternative approach is required to generate this type of mesh and analyse crack growth. The possibilities currently available are:

- use of mapping techniques to remesh a topologically fixed region in space which may contain the crack growth
- advancement of the crack through a hex mesh
- automated volume meshing with tet elements surrounding hex elements at the crack front

The remeshing of a fixed region in space was discussed in by Cook et al in Ref. 12. This method uses mapping techniques to replace a standard element in an uncracked mesh with groups of brick elements, or a “crack-block”, that contains a section of crack front. The original method only allowed growth to occur within the volume occupied by the original element of the uncracked mesh. The current implementation in ZENCRACK allows greater flexibility by:

- shifting of the boundaries of the crack-blocks
- relaxing elements surrounding the crack-blocks
- transferring crack-blocks from one location to another
- allowing use of “large” crack-blocks to increase the volume in which growth may occur

These methods are based on an existing uncracked finite element mesh. They allow loading and boundary conditions to be updated as the crack is incorporated and advanced through the mesh. The methods are general and can be used with any finite element program if the interfacing code is written to interpret an uncracked mesh and write a cracked mesh.

The use of automated volume meshing with tets is more likely to be an implementation linked to a specific pre-processor since the analysis would be based on a geometry definition rather than an uncracked f.e. mesh. The closest commercial implementation to this type of approach will be available in Abaqus/CAE version 6.5, released towards the end of 2004. That implementation, tied to Abaqus/Standard, is for one-off crack positions and provides no growth capability.

An example of the shift / relax / transfer techniques mentioned above is shown in a side view of a symmetry model for a SEN specimen in Figure 5. (Note that this is a side view of a 3D mesh). The first picture shows a regular uncracked mesh. In the second picture a crack-block is inserted using the original methodology of Ref. 12. It is clear that some element distortion exists within the crack-block and that there is a limit to the crack positions and growth that could be analysed. In the third picture the boundary of the target crack-block element is adjusted in order to produce well-conditioned elements within the crack-block region. This leads to some distortion outside the crack-block that is resolved by the application of a relaxation algorithm as shown in the fourth picture. These methods allow the crack to grow to new sizes by shifting the boundary by increasing amounts. Eventually the crack-block is transferred to the next element as shown in the last 2 pictures. In this way the crack front can travel through a mesh.

An example of this technique applied to a pin-loaded lug is shown in Figure 6. This problem requires contact conditions between the pin and lug in the f.e. analysis. Maximum principal stress in the uncracked mesh is used to determine an initial defect location. The lug has 4 elements through the thickness and hence 8 elements are replaced by crack-blocks (four on each side of the crack). In the mesh of the initial defect it is clear that the elements into which the crack-blocks are inserted have been reduced in size. This is done to attempt to minimise the element distortion at the crack front. As a consequence the

surrounding elements are relaxed in order to reduce distortion in that region. As the crack grows the element boundaries are shifted to accommodate the increasing crack size. When it is unfeasible to shift the boundaries any further, the crack-blocks change location and move into the second row of elements. This procedure continues as the crack grows, allowing the defect to move through the mesh. As can be seen in Figure 6, the crack that develops is determined by the loading and materials data and no assumptions are required about the crack following particular element boundaries. Once started, this analysis is completed fully automatically.

It is not always obvious how such meshing methods can be used. For example, a tricky topological problem is presented by a crack between two plates, as shown in Figure 7. In this case the crack ends at the vertical surfaces of plate 1 but from the point of view of plate 2, there is additional material beyond the end of the crack front. Since the detail of the elements surrounding the crack cannot be extended into plate 2, the modelling approach is to cut the mesh to allow dissimilar meshing within plate 2. Tying constraints can automatically be generated to join the two mesh parts. (or glued contact, depending upon the f.e. code). This technique works well provided care is taken in defining the mesh densities (in order to obtain a consistent master / slave definition across the tying interface).

In the general case non-planar growth is possible. To make this non-planar growth possible, the crack face history is stored as an array of triangular facets. The nodes on the crack faces for the latest advanced crack position are mapped onto the history of the crack face and internal nodes are mapped with the crack face taken into account. This is demonstrated by two examples with mixed mode loading:

- The first example is growth of a 43 degree slanting crack in a plate under cyclic axial tension. This is based on a test case for which experimental data is published in Ref. 13. As with any numerical method there are always many ways to carry out the discretisation. In this example it is not difficult to imagine that the crack will grow towards the edges of the plate. The mesh is therefore set up to take this into account, as shown in Figure 8. This may be viewed by some as a form of cheating, but the finite element method is simply a tool – it is always the case that the mesh should be adapted to suit the nuances of a particular problem and crack growth prediction is no different to any other type of analysis in that respect.

The crack face triangular facets that are defined by Zencrack for the initial crack plus those calculated during the analysis are shown in Figure 9. The growth is compared against experimental results in Figure 10 and Figure 11. It is clear that there is excellent agreement in both the calculated growth direction and cycle history.

- The second example is a 4-point bend specimen containing a circular hole and a through thickness crack. This is based on a test case for which experimental data is published in Ref. 14. The reference also

contains a 2D plane strain calculation of the crack path. For this example it is possible to compare the crack paths but not the cycle count (due to insufficient details available in the reference).

The top picture in Figure 12 shows the uncracked model with the offset hole. The remaining pictures show the initial through thickness crack and the meshing at three stages during the growth prediction. This example shows how the crack-blocks are rotated to try to keep them sensibly aligned with the crack front, whilst the surrounding elements are relaxed. Figure 13 shows the calculated crack profiles superimposed on the uncracked mesh. It is clear that a significant amount of element shifting and relaxation is required to complete this analysis. Figure 14 shows the agreement between the Zencrack prediction and the reference.

The examples shown so far demonstrate non-planar growth for cracks that have mild curvature along the crack front. Even though the algorithms are general, it could be argued that a 2D analysis may have been sufficient in these cases. A further class of problems, however, are those for which there may be a significant effect of crack shape development. Two such examples are the interaction of two defects and the effect of residual stress on crack shape development. These cases require a 3D analysis even though the growth may be planar:

- Ref. 15 describes experimental results for the interaction of two co-planar surface defects in a number of fatigue-loaded three point bend tests. Representative defects have been analysed to demonstrate the potential for crack interaction within a finite element context. The analysis is conducted in two parts: growth of the separate defects followed by growth of the combined defect. It is not possible to model the moment of joining of the two defects due to issues of element and mesh definition. Instead, a small “jump” is assumed to occur instantaneously and the initial joined configuration is estimated based on the analysis of the separate defects. Meshes and growth profiles from the analysis of the two different sized initial defects are shown in Figure 16.
- The effect of residual stress on crack shape development is demonstrated by an example of growth of a corner crack in a square bar under cyclic end load. The bar is shot-peened on one face as shown in the mesh plots in Figure 17. The effect of this surface treatment is to introduce a compressive residual stress just below the treated surface, as shown in Figure 18. After a small number of load cycles this residual stress distribution exhibits elastic shakedown. Hence the application of a percentage of the full distribution is used to investigate it's effect on crack shape development and life of the component. A variety of analyses were conducted to investigate elliptic and circular starter cracks. The mesh plots in Figure 17 show a typical elliptic crack. The results in Figure 19 and Figure 20 are for a typical corner crack. Figure 19 compares the analysis results from Zencrack and Afgrow with no residuals stress and with 40% residual stress. There is close agreement for the case with no residual stress but a significant difference when the residual stress is

included. A major reason for this is that AFGROW does not take account of the shape development of the crack. As can be seen in Figure 20 the growth differs considerably from a circular shape when residual stress is included. Figure 21 shows the profiles for a crack depth of about 0.35mm for 0%, 30% and 40% residual stress along with the cycles to achieve this position. It is clear from these results that residual stress has a dramatic effect on crack shape development and the life.

SUMMARY

Commercial software is able today to calculate fracture parameters for even the most difficult of problems. Thermal load and body forces may be included although the inclusion of general initial stress presents greater difficulty.

The boundary element and finite element methods have been shown to be versatile tools for use in modelling crack advancement. Generally this is undertaken by specialised software that uses the structural solution as a means of calculating fracture parameters.

Generalised algorithms are available that taken into account crack shape development and non-planar crack growth.

The combined effect of shape development and non-planar growth gives the most difficult type of growth prediction. In fact, experimental data for such cases is few and far between. However, the general approaches of commercial software can tackle even these most difficult of problems.

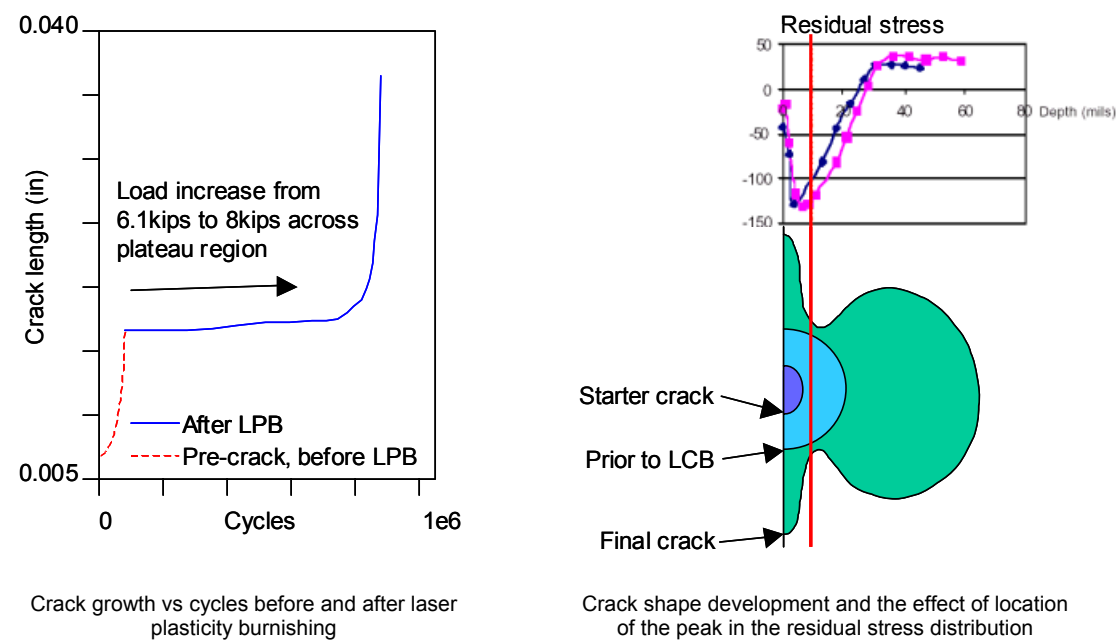
The possibility of including non-linear effects such as crack face contact and plasticity into a finite element analysis mean that complex crack analysis is possible. However, the results from such analyses must be used carefully in terms of crack propagation.

There are areas where difficulties remain. For example:

- The attractiveness of the simpler meshing of the boundary element approach placing a linear elastic restriction on the analysis.
- The potentially long run-time of repeated finite element analyses of large models may make some analyses prohibitive.
- The difficulties associated with the calculation of fracture mechanics parameters for generalised residual stresses.

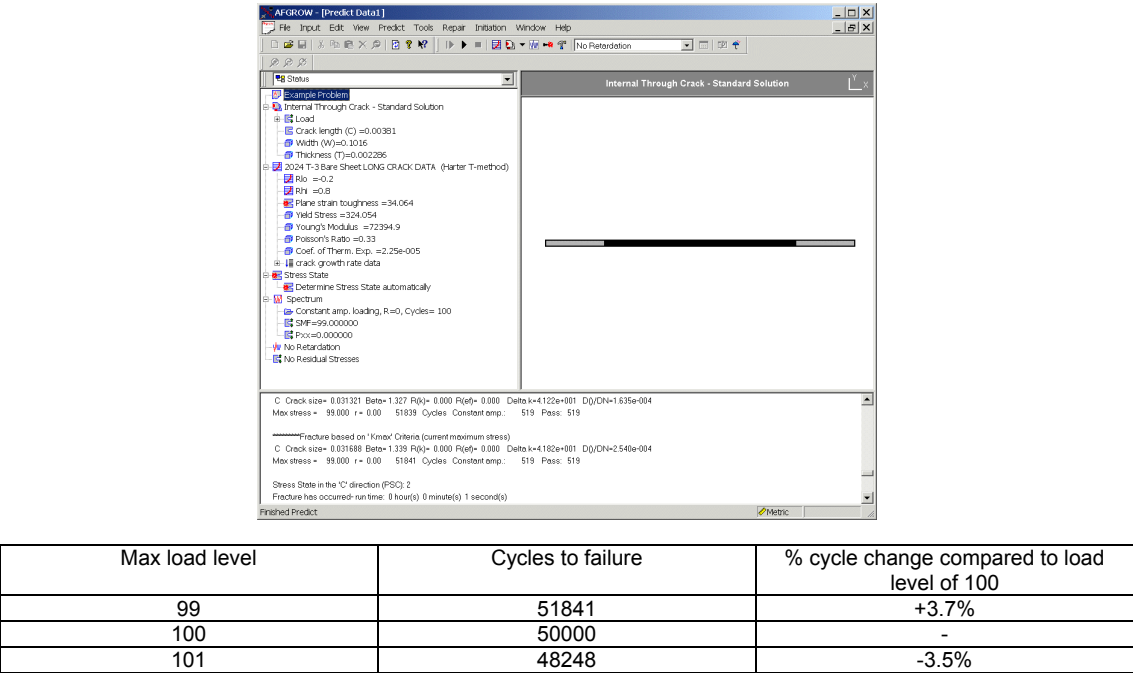
Further advances in both software and hardware will produce greater versatility in the solutions that can be provided in the future.

FIGURES



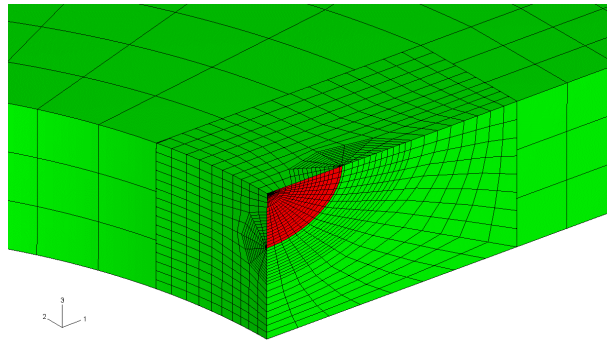
(Schematics based on figures in Ref. 1)

Figure 1

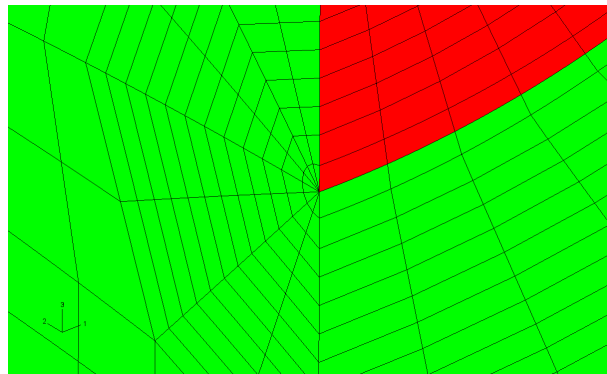


Effect of $\pm 1\%$ change in load level on life prediction calculated using Afgrow

Figure 2

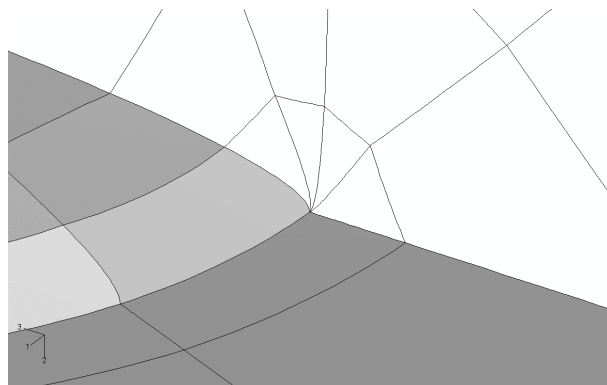


Typical f.e. mesh with element rings around an elliptic section of crack front (crack face shown in red)

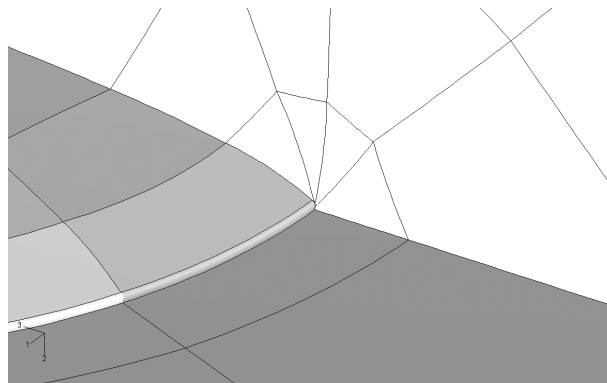


Close-up showing collapsed (wedge-shaped) elements at the crack front

Figure 3



Quarter-point nodes and a single node at each crack front location – suitable for LEFM



Midside nodes and multiple (retained) nodes at each crack front position – suitable for EPFM

Figure 4

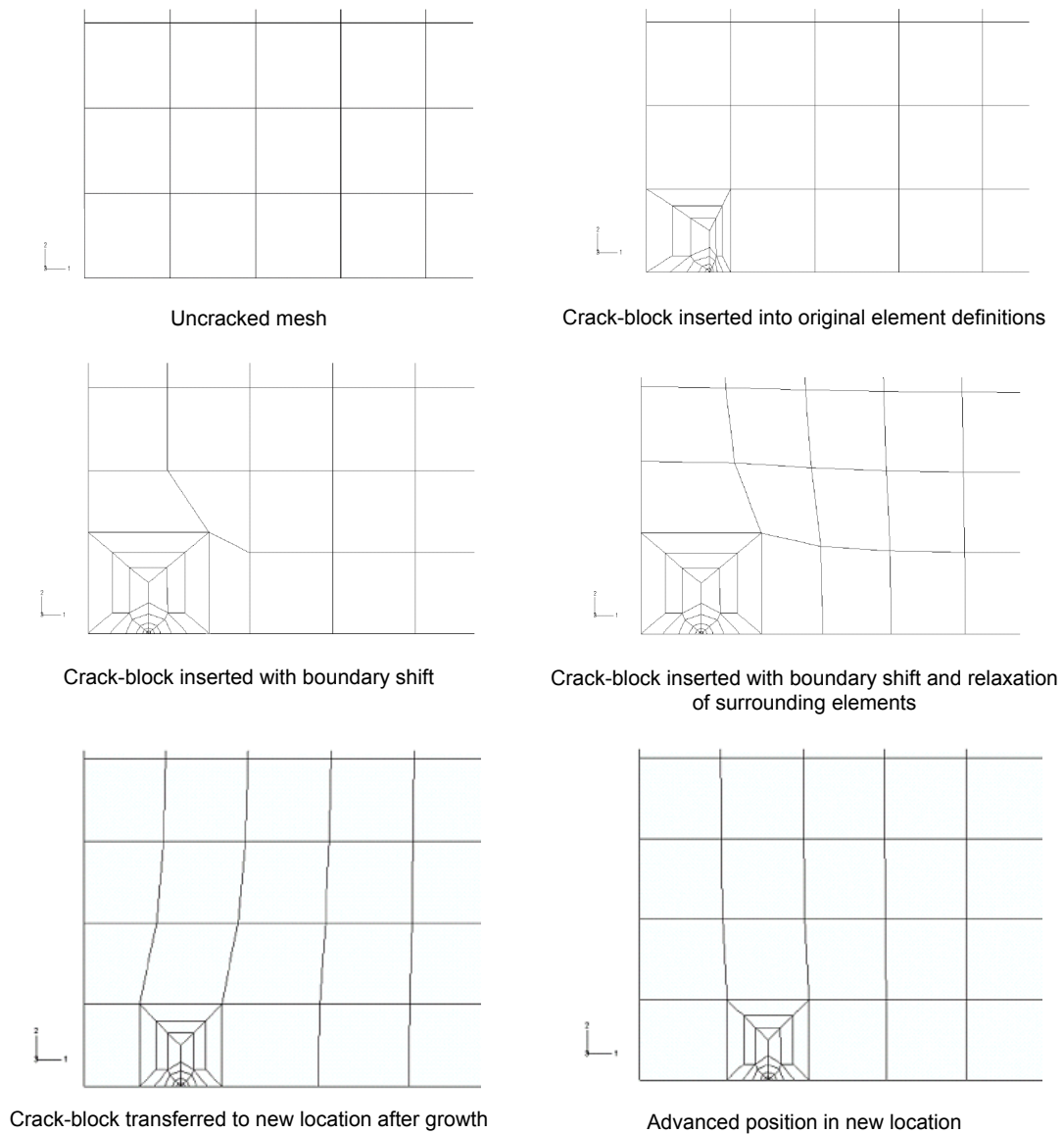
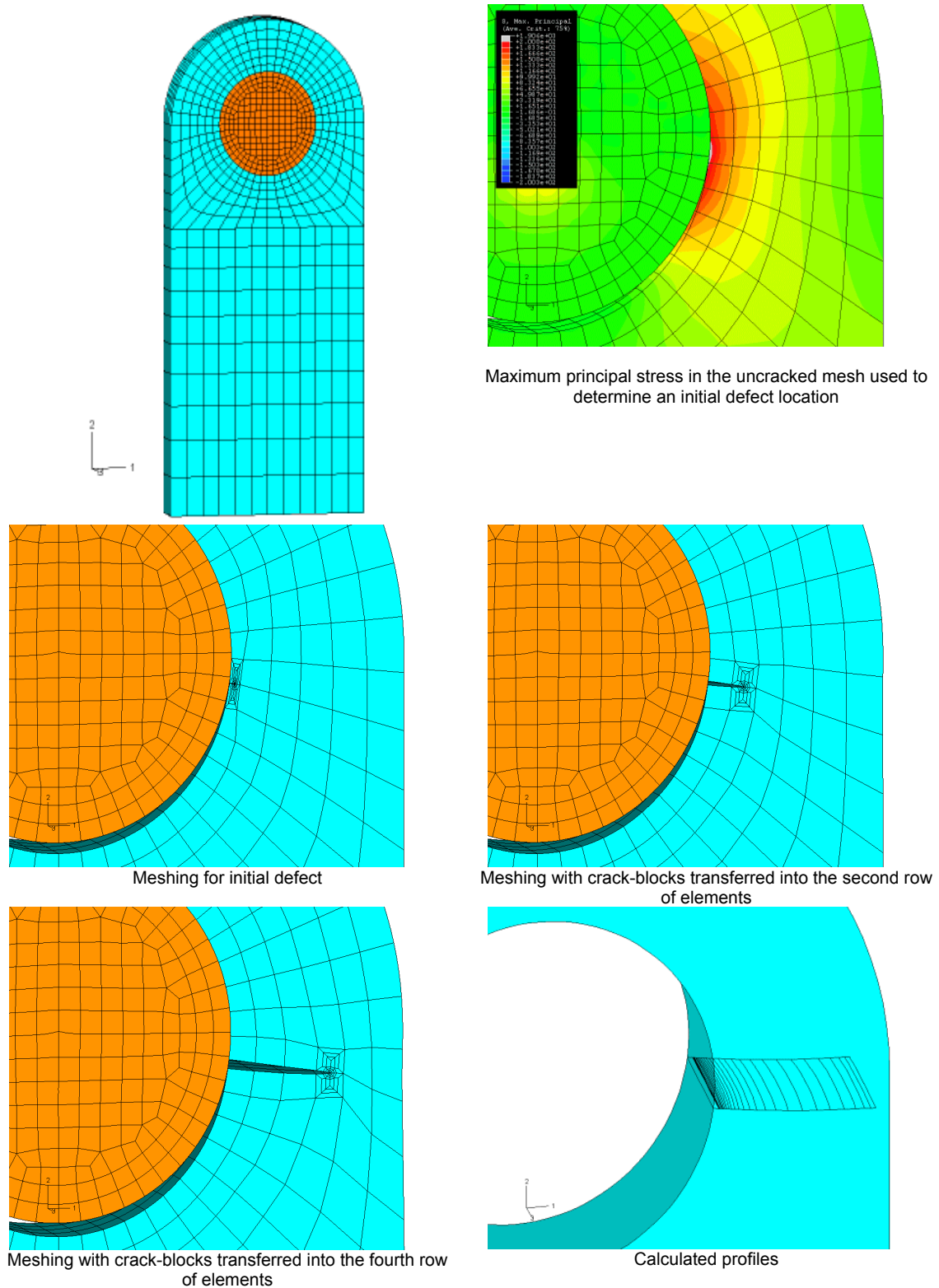
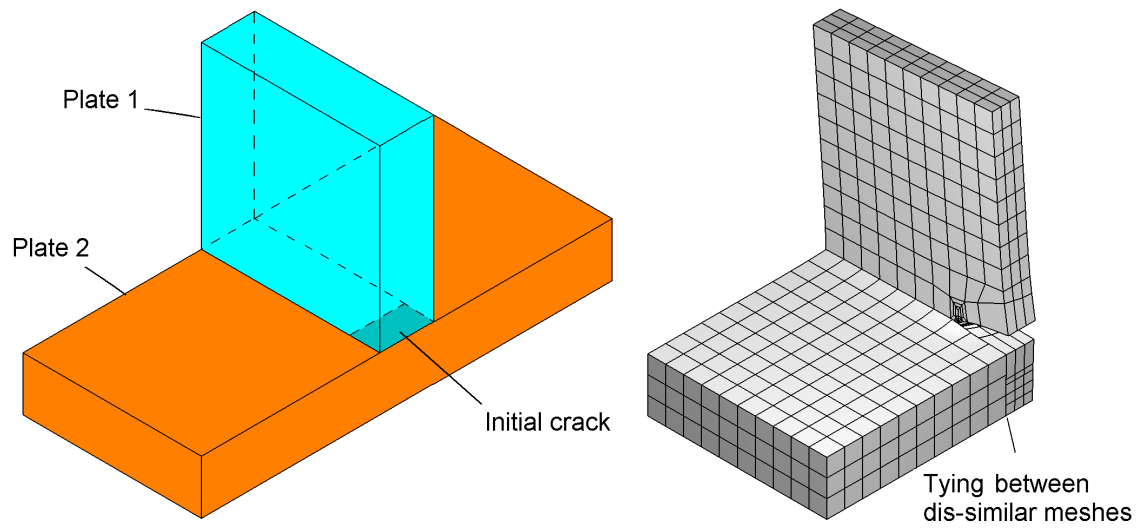


Figure 5

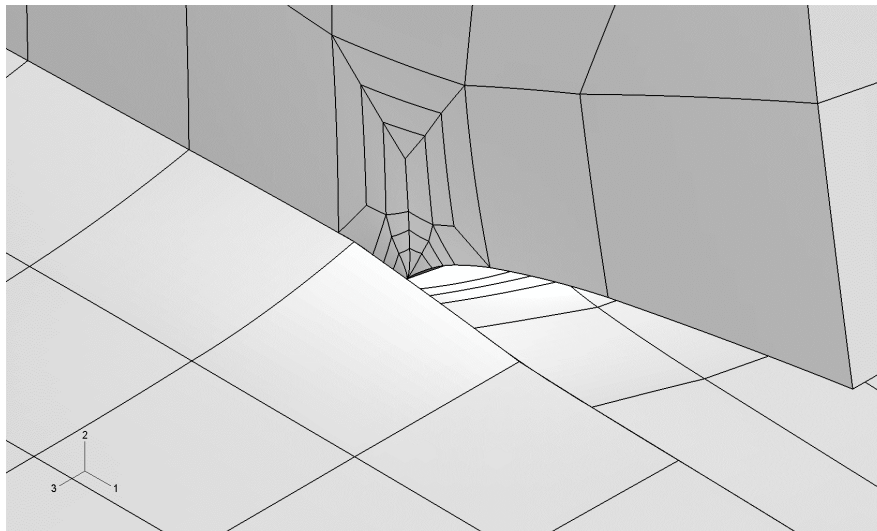


An initial through crack grows through the lug. Note the crack path is independent of element boundaries in the uncracked mesh and that the mesh surrounding the defect is modified as the crack advances. The final picture shows the calculated crack profiles.

Figure 6



Example of a crack at the intersection of two plates using a symmetry f.e. model with tying of dis-similar mesh regions



Close-up of tied region and the end of the crack front

Figure 7

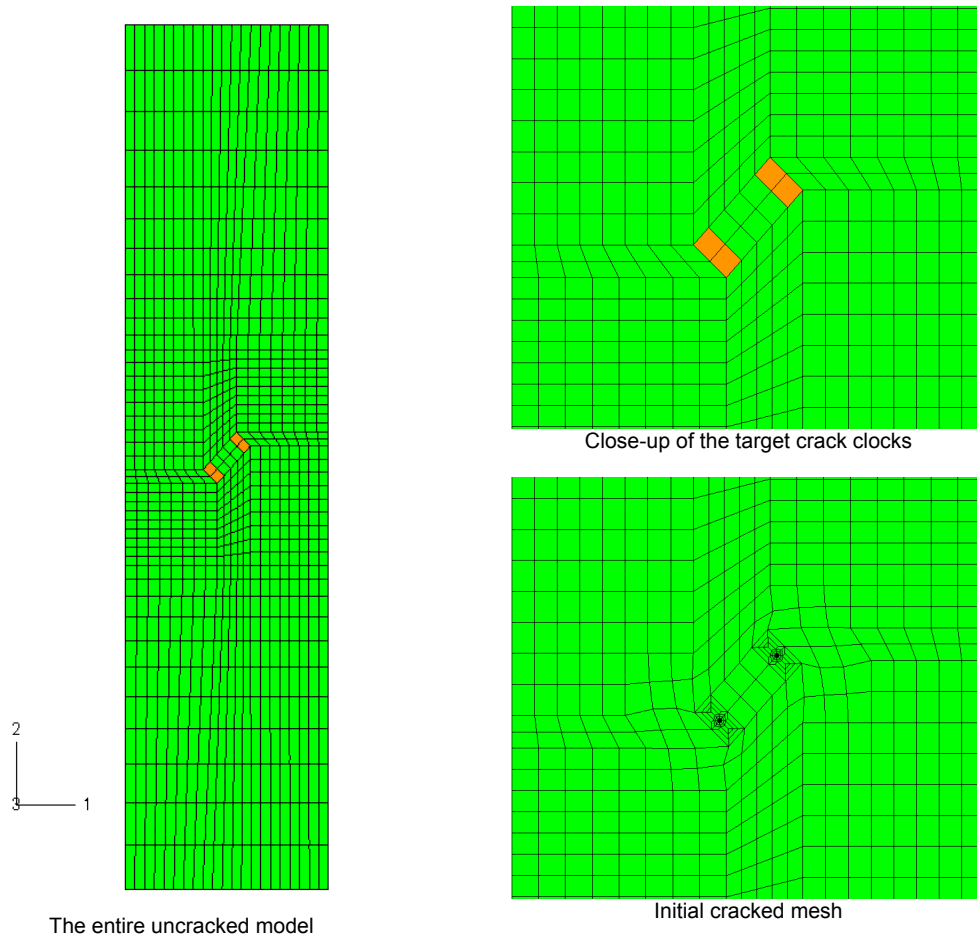


Figure 8

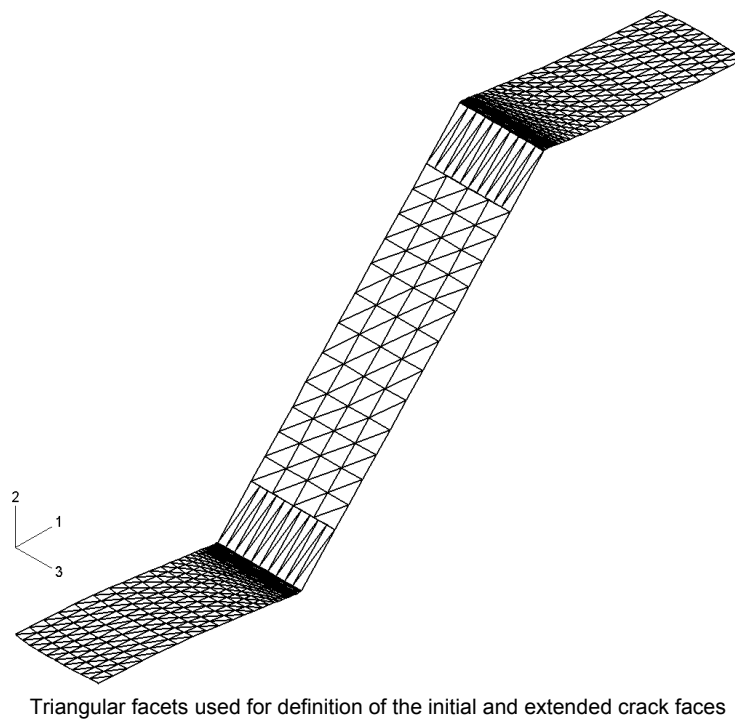


Figure 9

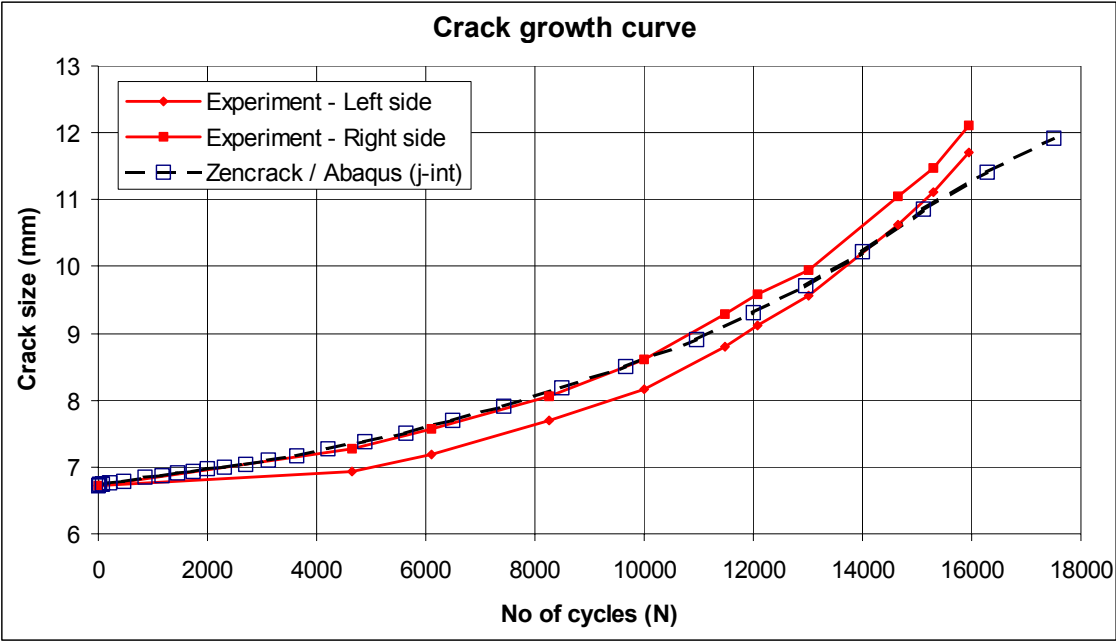


Figure 10

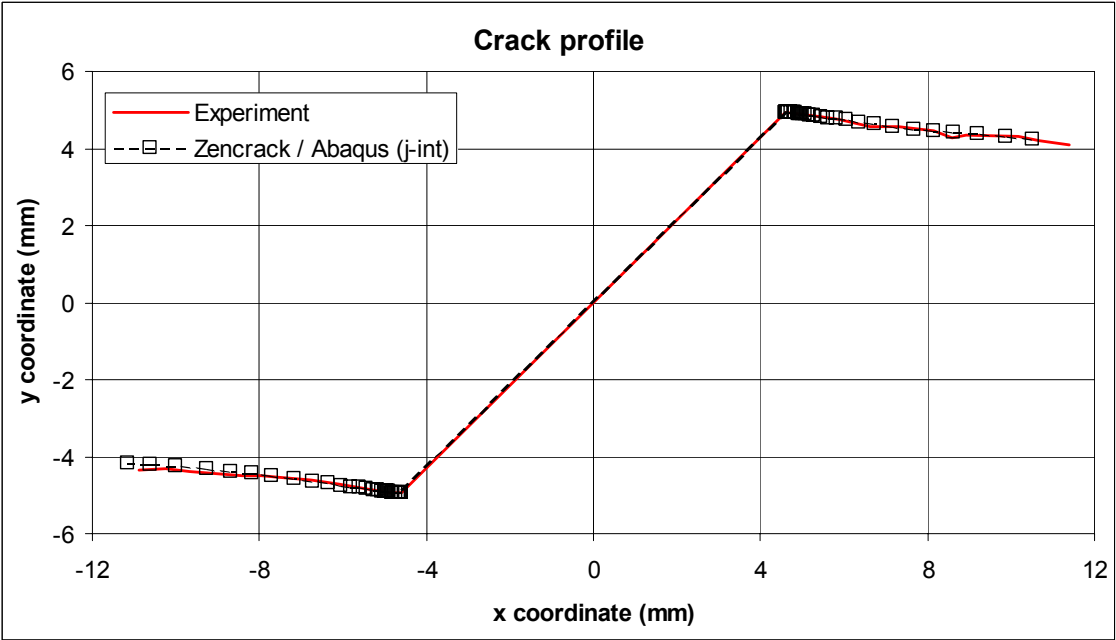


Figure 11

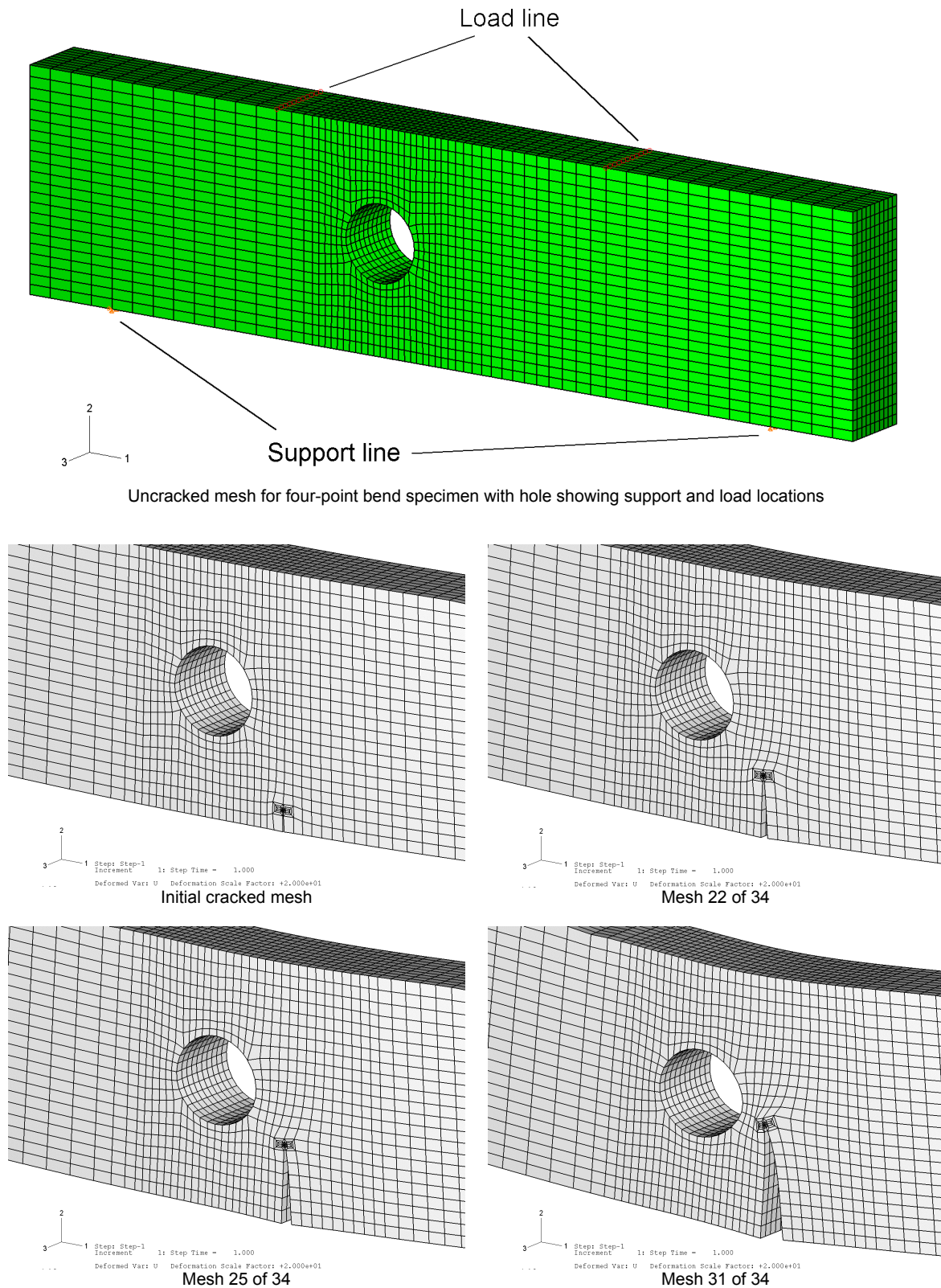
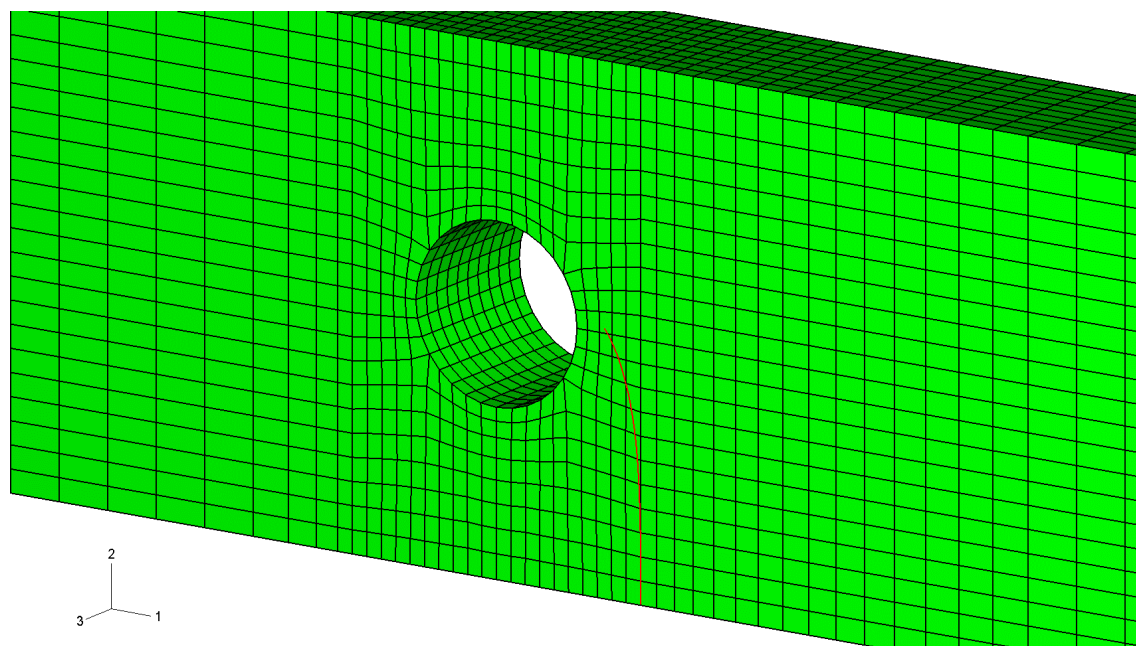
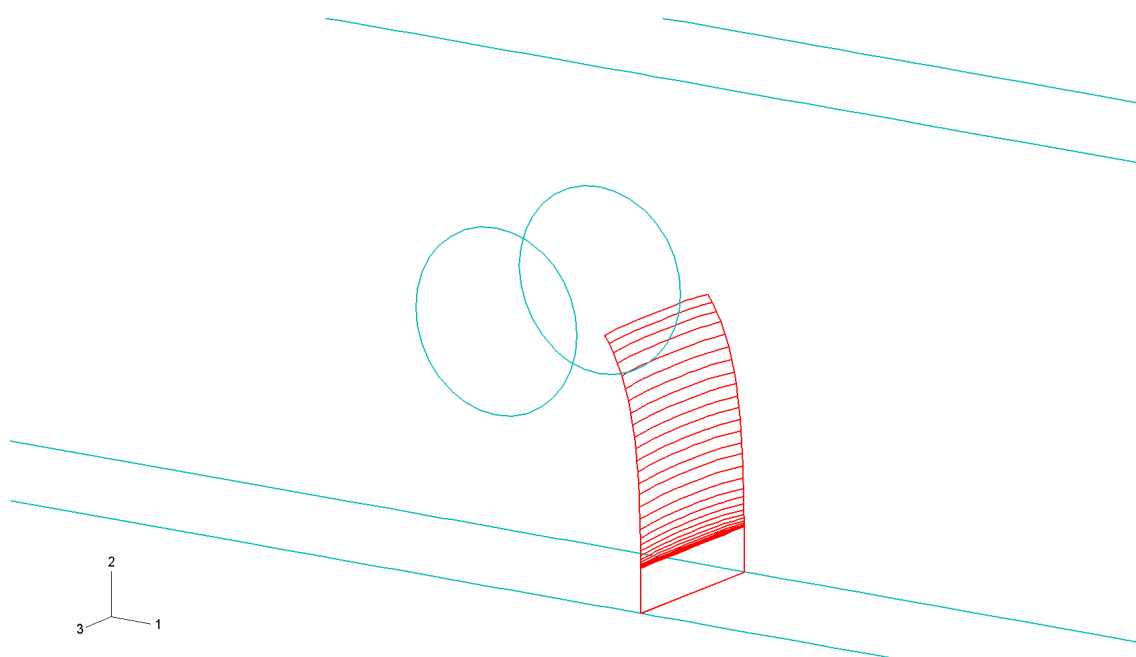


Figure 12



Calculated profiles superimposed on initial mesh



Calculated profiles

Figure 13

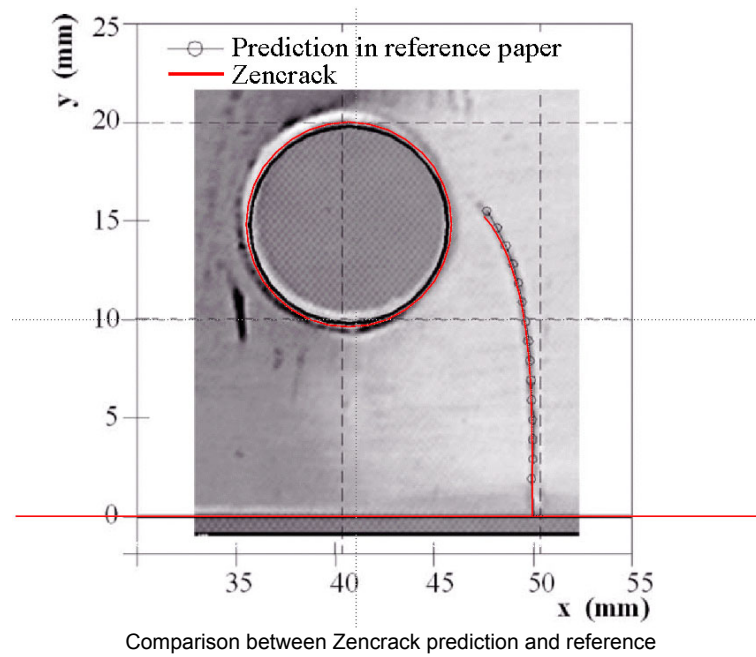
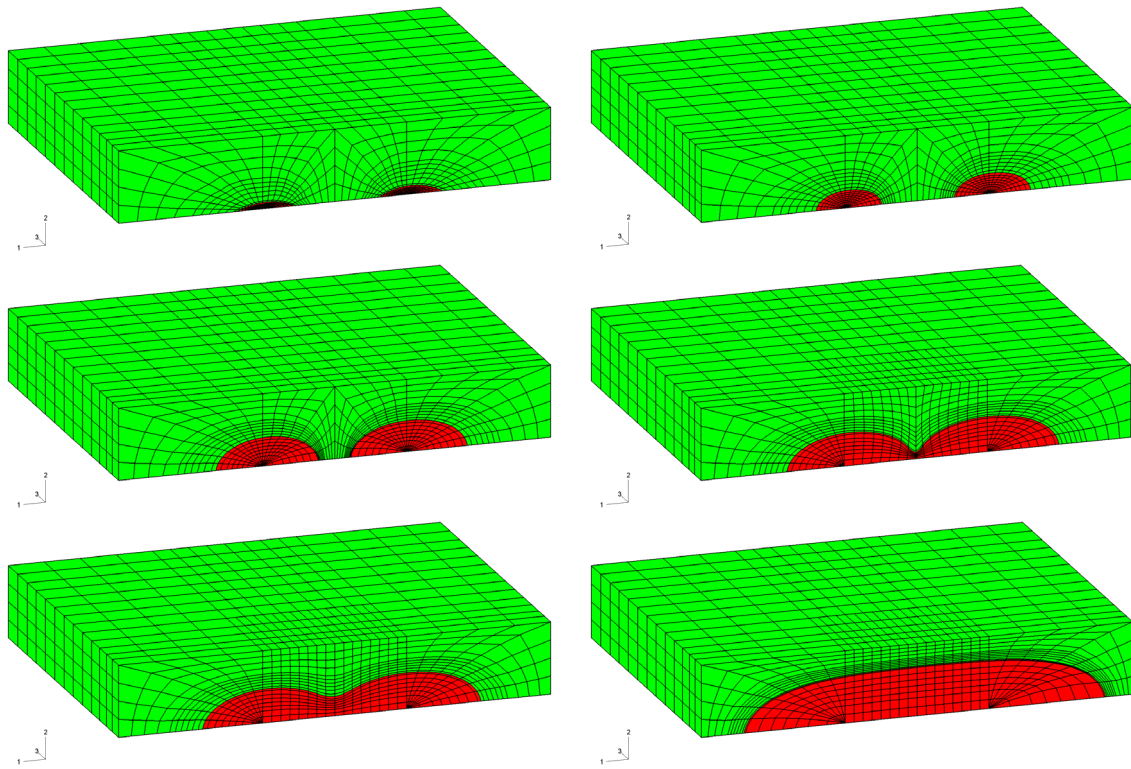
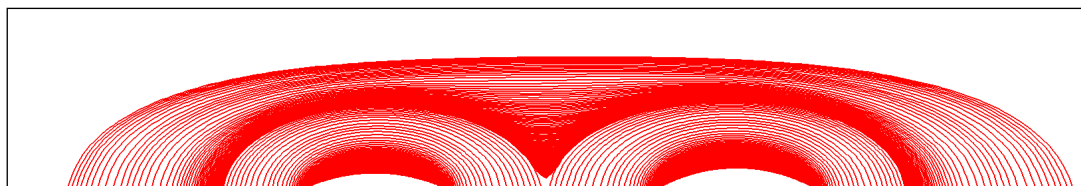


Figure 14



Initial and final cracked meshes with 4 intermediate stages

Figure 15



Calculated crack profiles

Figure 16

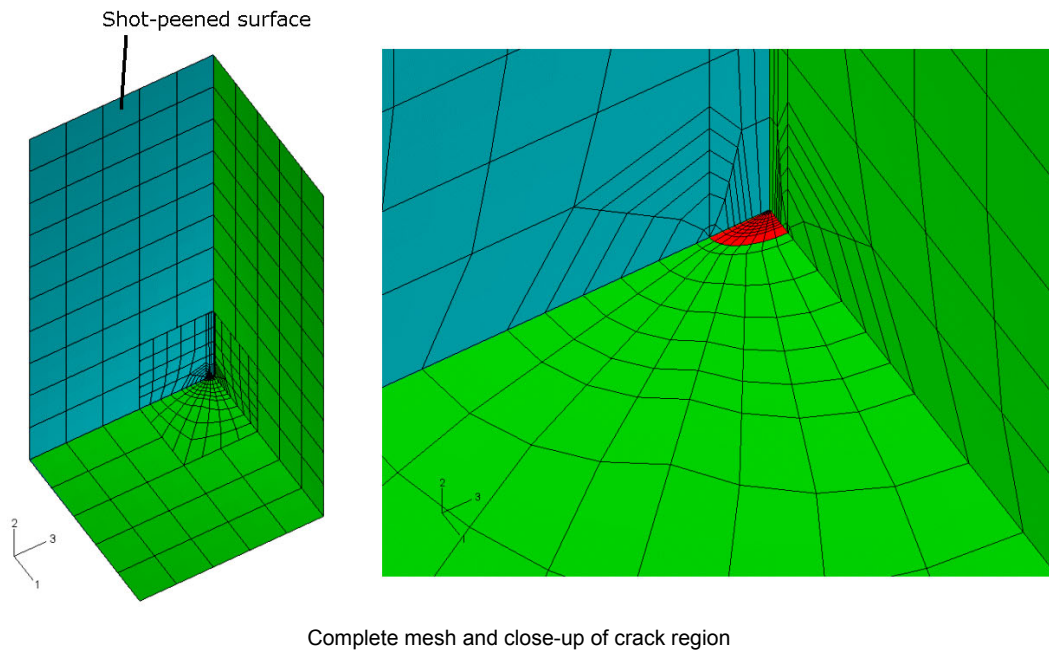


Figure 17

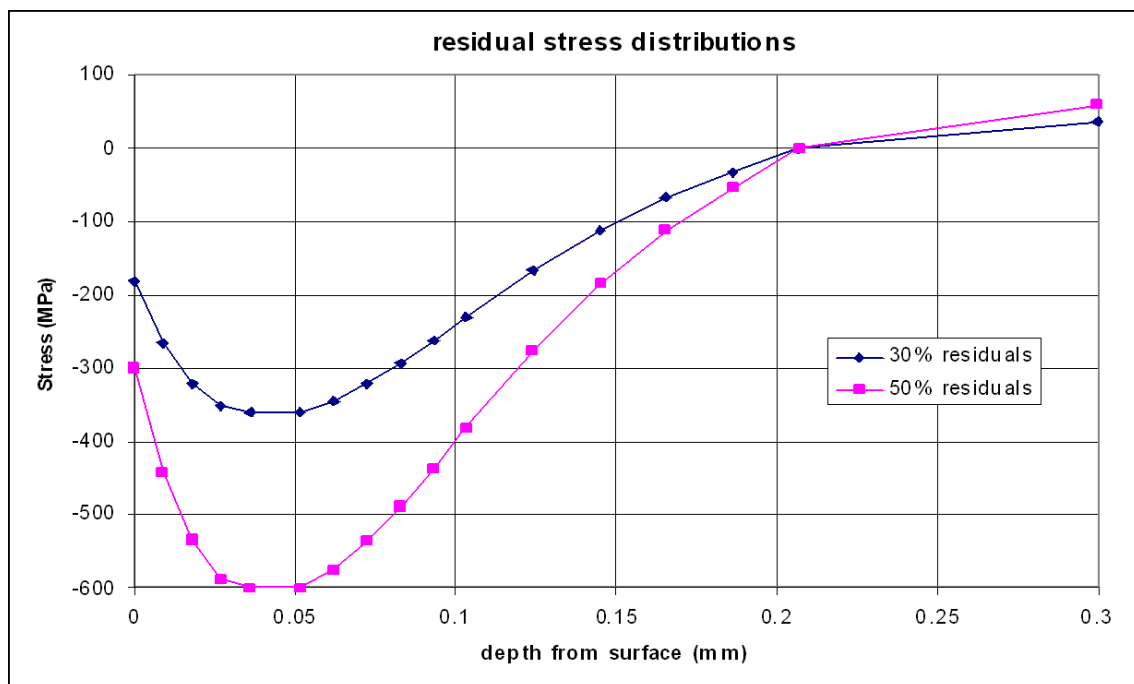
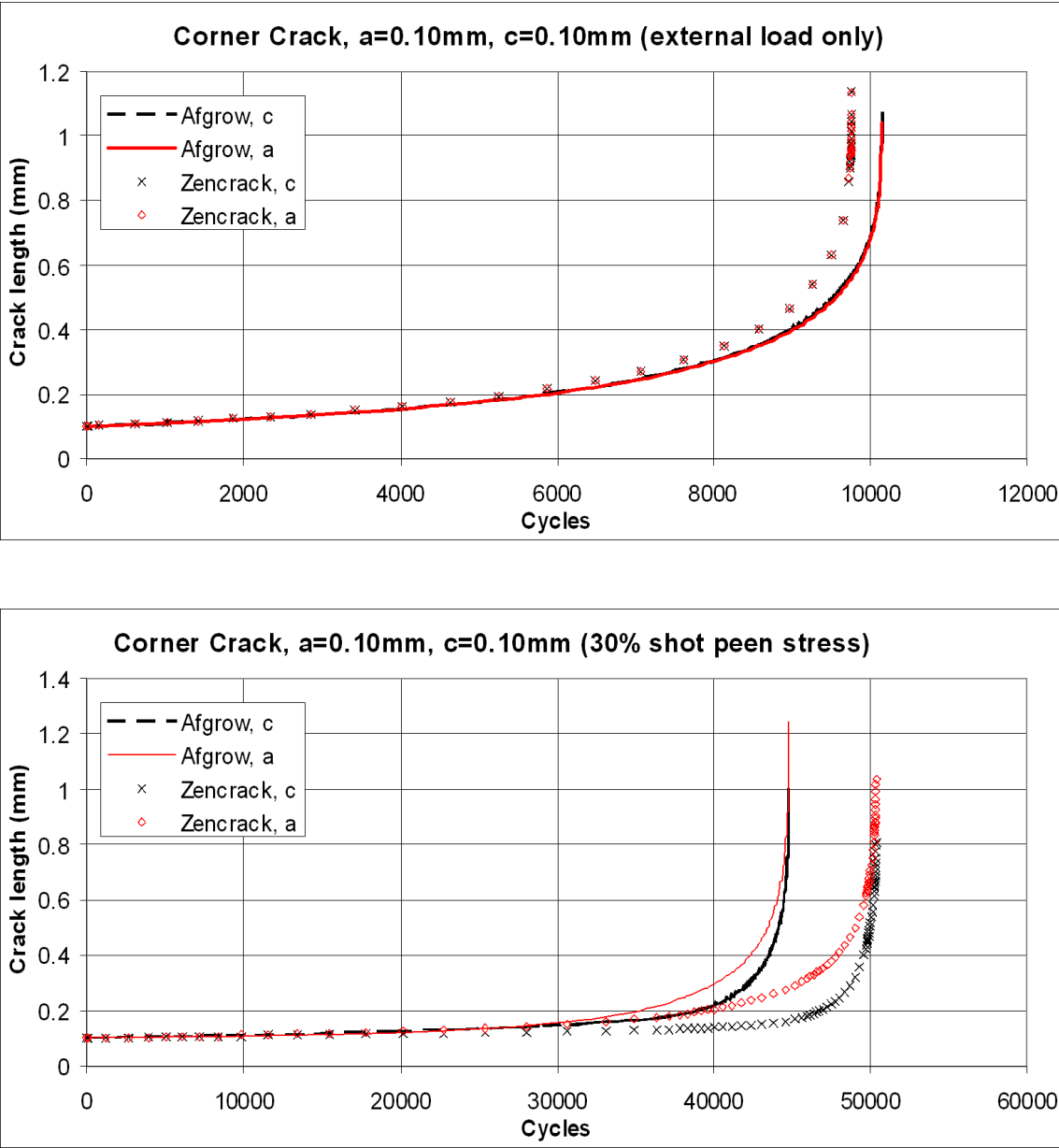
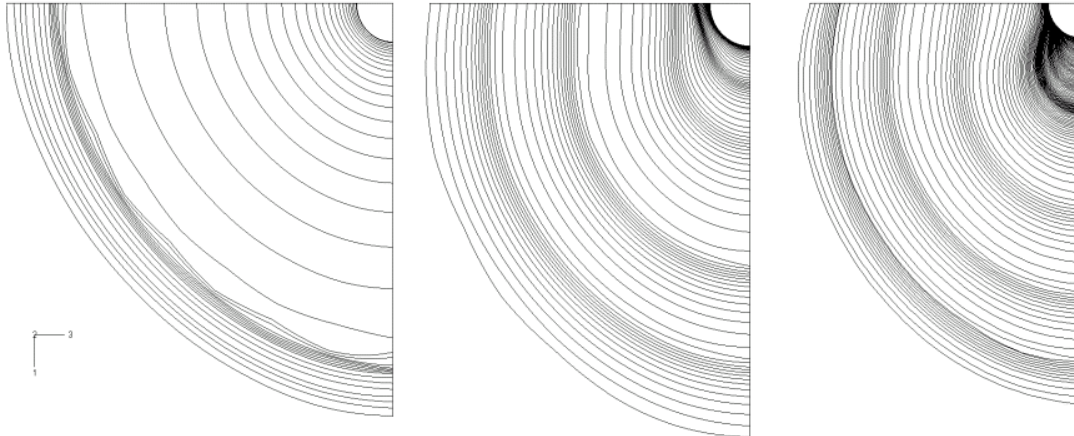


Figure 18



Comparison between Zencrack and Afgrow of growth on the treated and untreated surfaces (c and a respectively) for 0% (top) and 30% (bottom) residual stress applied with external cyclic load

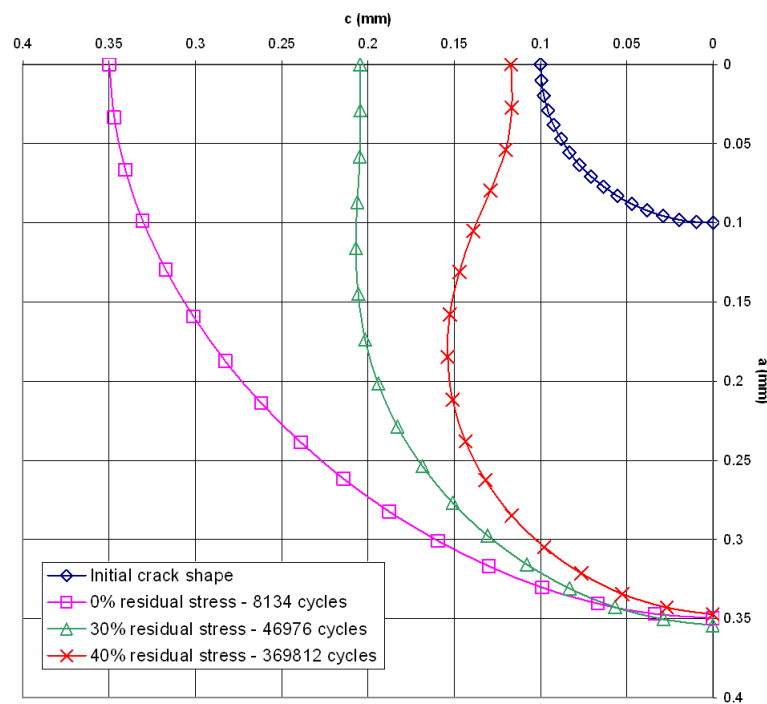
Figure 19



Profiles for 0% (left), 30% (middle) and 40% (right) residual stress applied with external cyclic load

Figure 20

Profiles for a of around 0.35mm



Profiles and number of cycles for crack size of about 0.35mm along the untreated surface

Figure 21

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