SIMULATION OF 3D NON-PLANAR CRACK PROPAGATION

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SUMMARY

Simulation of 3D non-planar crack propagation behaviour is fundamental to product development, assessment and management of the residual life of components. It also plays an important role in performing forensic studies to understand and establish the true loading conditions that led to accidental or premature failure of a structure. This is achieved by generating computer simulations of 3D fracture surfaces which match observed failure modes.

The authors have been involved in developing a software simulation tool for these purposes and have interfaced it to a number of established commercial finite element codes. This enables all the inherent facilities of a particular finite element code to be used to full advantage without having to re-invest in or 're-learn' a finite element analysis tool. The man years of development within these finite element codes ensures that many mechanical, thermo-mechanical and contact problems of cracked components can be solved with relative ease once the cracked mesh is available. In this respect the developed software works with any existing pre-processor to allow cracks to be inserted into an uncracked component.

The code is general in its implementation and allows cracks to be placed at arbitrary locations within a component. The full effect of mode-mixed loading is accounted for in the analysis and this allows for non-planar crack growth prediction. Surface and sub-surface defects may be modelled and the resulting crack growth calculated under complex spectrum fatigue loading or time dependent load conditions.

Examples are highlighted of the analysis of surface cracks in rotating shafts and of crack growth in structures which experience considerable contact loading. In such cases the mixed mode effects can vary constantly through the load cycle and a new method is described to handle these non-proportional load scenarios.

Keywords: Crack growth simulation, 3D, non-planar, j-integral, mixed mode loading, non-proportional loading.

1: INTRODUCTION

The prediction of crack advancement through a structure under time dependent or fatigue based loading in not a new requirement in the simulation world. Defects arising from material discontinuity or imperfection, applied loading and geometric factors can all contribute to product failure. The areas of application of the simulation methods are continually extending and pushing the boundaries of the theoretical background that is available. Simulation tools are themselves helping to increase this theoretical understanding by application within research organisations. In the commercial environment the challenge is often to answer a simple question – why did the component fail?

In addressing these issues the use of 3D finite element analysis provides many advantages over other approaches. The “real world” does not conform to the mode I scenarios that can often be addressed by simplified equation-based assumptions. The development of complex fracture planes seen in real components can only be simulated in software by the application of appropriate modelling techniques and mixed mode fracture mechanics principles. The finite element method has the benefit of general applicability and can be used in many ways for this type of application e.g.:

- improvement in product design by minimising risk of cracking
- forensic investigation and verification of a failure mode
- investigation of sub-surface defect development under moving contact (Hertzian) loads
- simulation of tests for new material development e.g. CT tests for epoxy resin materials
- thermo-mechanical fatigue crack growth assessment with temperature dependent material data
- simulating crack growth in gear teeth

This paper describes some issues related to mixed mode 3D crack growth simulation and demonstrates how the finite element method can be used to address those issues through use of the commercial Zencrack code [1].

2: A BRIEF THEORETICAL BACKGROUND

It is well known that linear elastic fracture mechanics provides a method for describing the stress intensification at a crack front. Three modes of local behaviour are used to define the opening, sliding and tearing stress intensity factors, K_I, K_{II} and K_{III} respectively. These can in turn be combined to obtain an equivalent energy release rate term, Gequiv:

$$G_{equiv} = \frac{B}{E}\left(K_I^2 + K_{II}^2\right) + \frac{1}{2G}K_{III}^2$$

where:

$B = 1 - \nu^2$ for plane strain

$B = 1$ for plane stress

$G = \dfrac{E}{2(1+\nu)}$ G=shear modulus, E=Young's modulus, ν=Poisson ratio

Expressions exist to calculate the crack growth angle based upon the stress intensity factors. These expressions are derived from linear elastic theory and are strictly only valid for mode I and mode II (i.e. they assume K_{III}=0). A more general approach is to evaluate the direction of maximum energy release rate at a crack front point. This can be done by application of a series of virtual crack extensions that ultimately generate a growth angle at the crack front node that, in the general case, may be out-of-plane, as shown schematically in Figure 1.

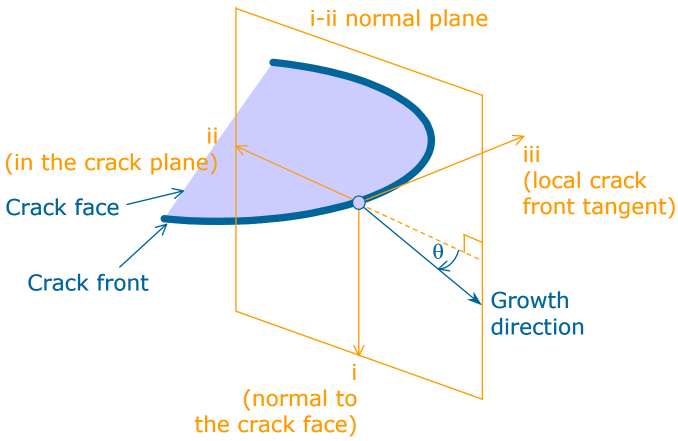


Figure 1: Local out-of-plane crack growth for a point on a 3D crack front.

In a fatigue crack growth scenario it is the range of stress intensity factor that dictates the damage coupled with other effects such as load sequence, frequency, temperature etc. The crack growth relationship, in the simplest form, is described by the Paris equation:

$$\frac{da}{dN} = C(\Delta K_I)^n$$

where:

a = crack size

N = number of fatigue cycles

C, n = material constants

ΔK_I = stress intensity factor range = $K_{I\max}$ - $K_{I\min}$

An extremely important point to note here is the fact that crack growth data is most often presented as a function of only the mode I stress intensity factor.

In real applications where mixed mode loading occurs, the crack will very often attempt to find a configuration in which mode I dominates. However, this may not always be the case and examples exist where cyclic mode II behaviour can be dominant. These examples generally occur in contact problems where a defect is acted upon by a compressive load which moves across a component surface e.g. roller supports, shaft / bearing Hertzian contact.

For these cases the crack growth data should strictly be provided to the analysis as a function of ΔK_{II}. However, such data is not usually available. The alternative is to use the mode I data and convert it into an energy equivalent. This, coupled with the energy release rate range, can then be used for crack growth calculations. This is the basis of the growth calculations within Zencrack [1].

This energy-based approach also addresses the difficulties of mixed mode effects that arise when the loading is non-proportional in nature. In such situations multiple loads types are not cycling together and their interaction produces an ever changing mixed mode effect through a single load cycle. The instantaneous direction of crack growth changes through the cycle and from this an overall growth direction for the cycle must be calculated for use in crack growth prediction. An example of this type of behaviour that is fairly commonplace is a rotating shaft which experiences tension and torsion loading. In the following section this type of specimen is used to demonstrate the approach to crack growth prediction for three load scenarios.

3: SURFACE CRACKED SHAFT UNDER TENSION AND TORSION LOAD

The example of a circular cross-section shaft under tension and torsion loading provides an insight into the way that mixed mode effects can be incorporated into an analysis. The example is based on a test specimen and is therefore readily understood. Loading is applied at the ends of the specimen with an initial straight surface crack at the specimen mid point. Three loading situations are presented:

- cyclic tension load without torsion
- cyclic tension and in-phase cyclic torsion
- constant tension and cyclic torsion i.e. a non-proportional load system

The uncracked model and a cross section at which the initial crack will be located are shown in Figure 2, along with the equivalent mesh plots for the initial straight crack. The specimen is steel and has a radius of 6mm. The initial straight crack has a maximum depth of 1mm.

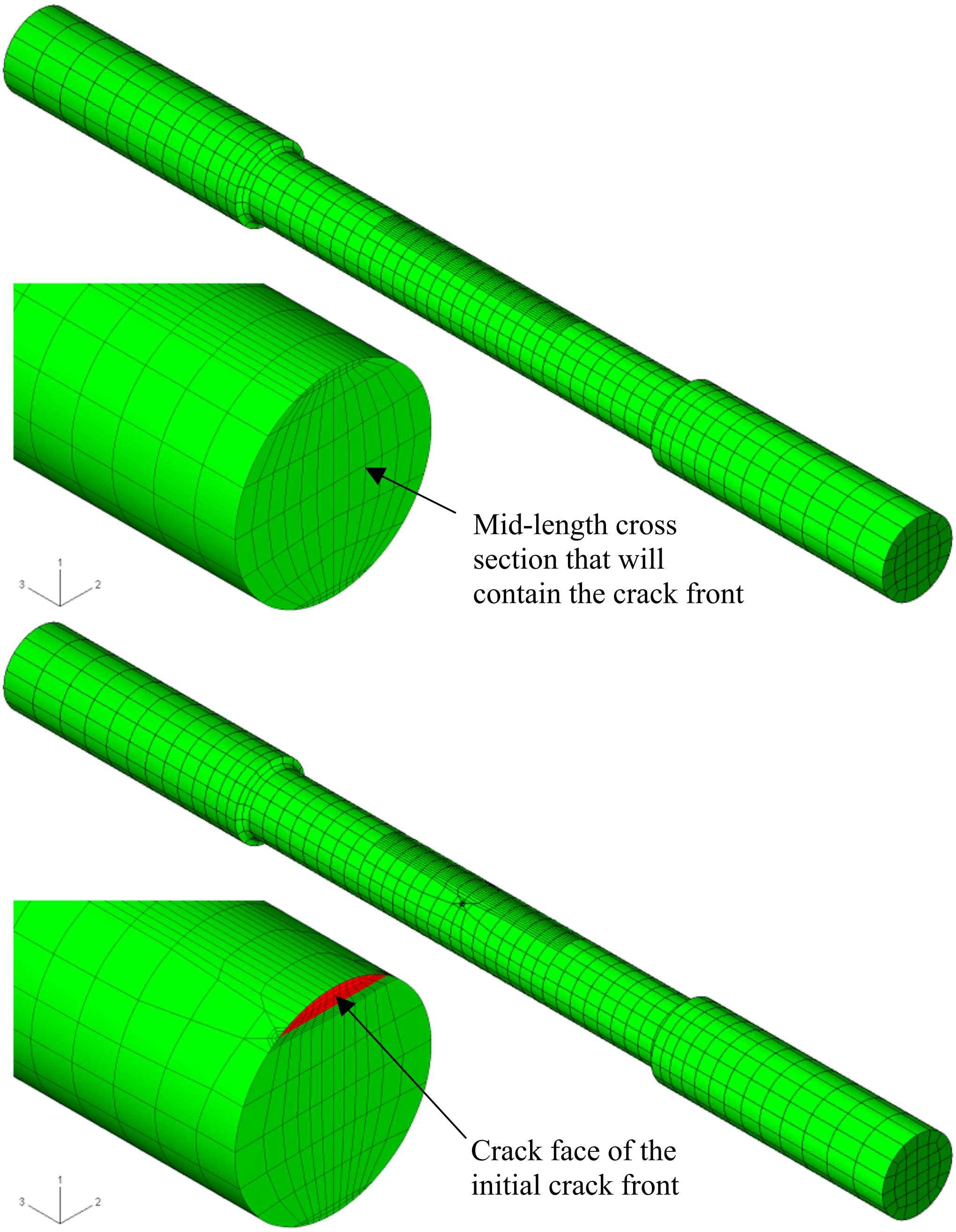


Figure 2: Uncracked (top) and initial cracked (bottom) meshes for the tension-torsion shaft analyses.

Under constant amplitude tension load there is no great difficulty in the fatigue crack growth analysis. If the component is analysed at a "unit" load level, then the results are readily scaled to calculate results at other load levels (as is done when analysing growth for a spectrum load). The behaviour is purely mode I and the crack advances in the initial crack plane. The initially straight crack front develops an in-plane curvature that would be difficult to calculate manually. Deformed mesh plots at two stages in the analysis are shown in Figure 3.

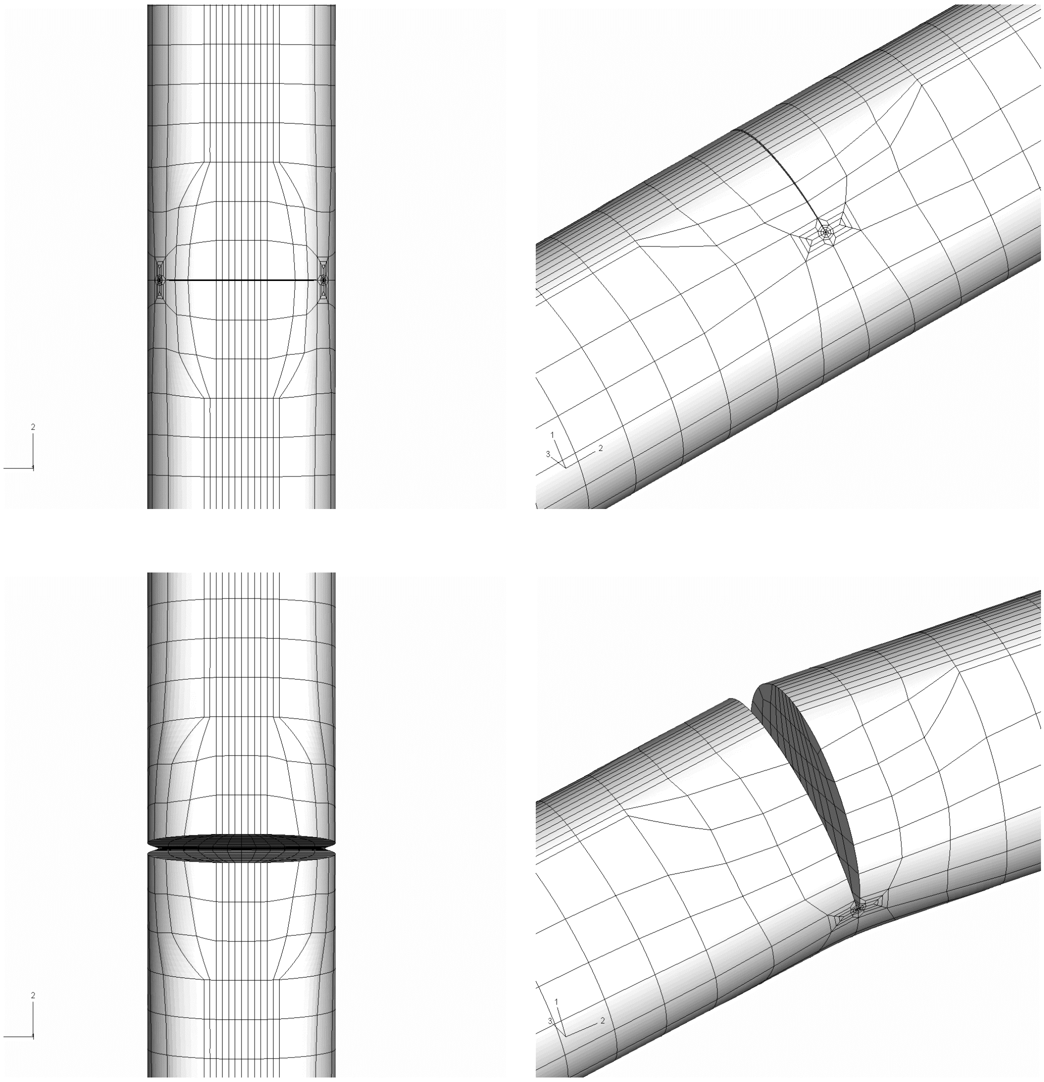


Figure 3: Two stages during the analysis for tension-only loading (deformations magnified).

The analysis becomes more interesting when torsion is added to the tension. In the first such example the loads are in phase. Because the loads are in phase, the growth direction at each crack front node remains the same through the load cycle. Therefore, it is again possible to analyse at a single load level and apply scaling to obtain results for other load levels. Deformed mesh plots at two stages in this analysis are shown in Figure 4.

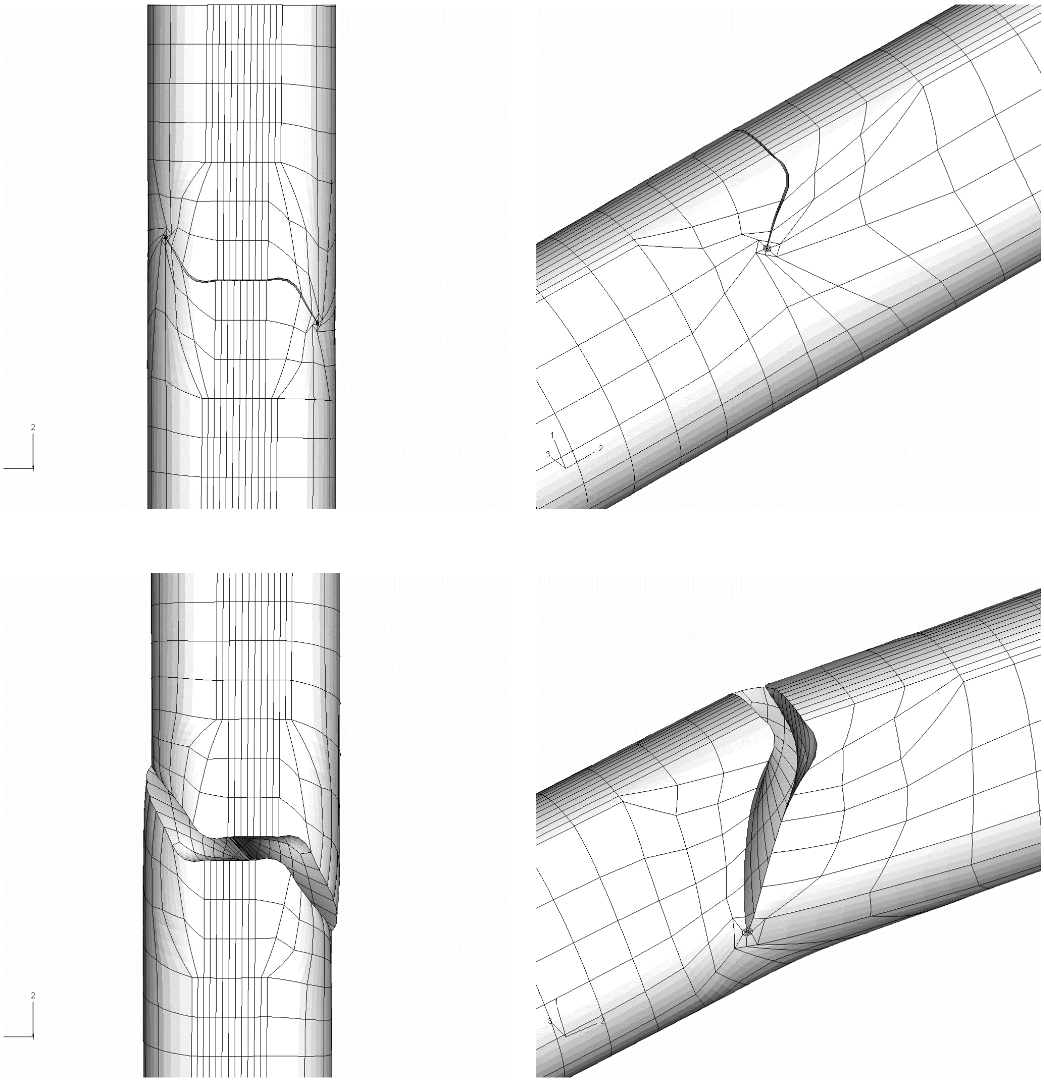


Figure 4: Two stages during the in-phase tension-torsion loading (deformations magnified).

The overall effect on the crack development when torsion is added can readily be seen in the deformed plots. The profiles showing the crack advancement through the section for the two cases are shown in Figure 5.

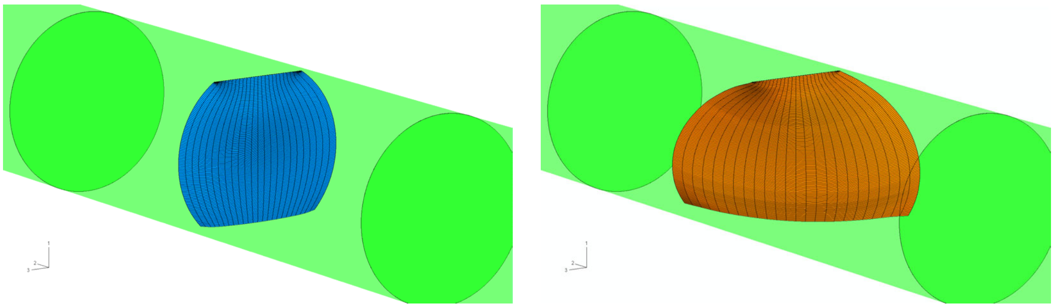


Figure 5: Comparison of tension-only and in-phase tension-torsion crack development.

The third load case to be considered is that of constant tension and cyclic torsion (0 to maximum). If we now consider the load level changes through the load cycle, it is clear that at any instant a crack front node will have a local instantaneous energy release rate and growth direction. Both the magnitude of the energy release rate and the growth direction will change through the load cycle. Figure 6 shows the energy release rate value and out-of-plane growth angle as a function of torsion load level for nodes A to G along the crack front (these node positions are from surface to mid-thickness, evenly spaced). In order to use this data in a crack advancement routine, it is necessary to extract an overall angle and energy release rate range for a single load cycle. The range is readily obtained for each node by extracting the minimum and maximum values through the cycle. The unit vector for the growth direction over the entire cycle is calculated for each node as:

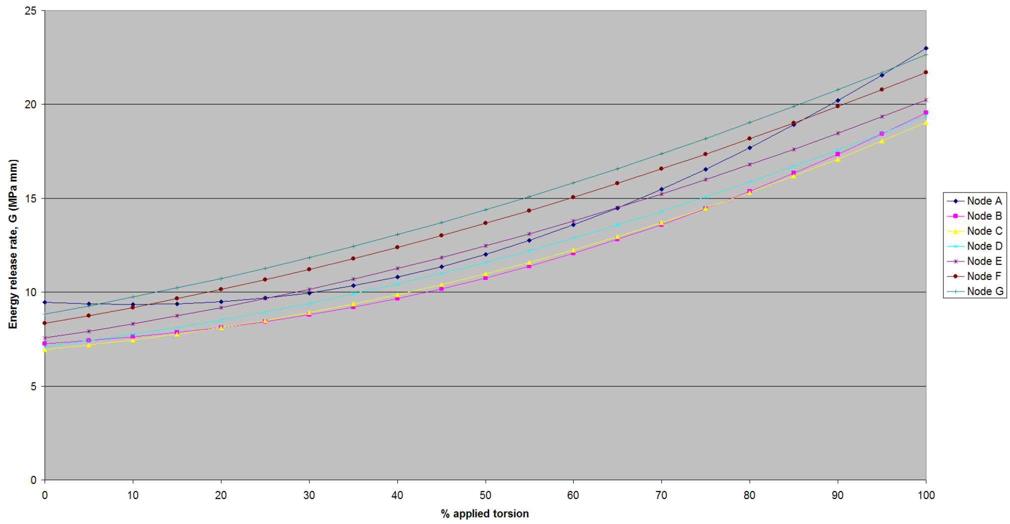
$$\hat{n} = \frac{\sum \sqrt{G_i}\ \hat{v}_i}{\left|\sum \sqrt{G_i}\ \hat{v}_i\right|}$$

where:

G_i is the instantaneous value of energy release rate magnitude

$\hat{v}_i$ is the instantaneous (unit) growth direction vector

At an advanced stage in the analysis (for which the mesh plot is shown in Figure 7), the crack has developed a non-planar shape and the plots of energy release rate magnitude and out-of-plane growth direction demonstrate the fairly complex behaviour underlying this apparently simple specimen. At the point in the load cycle when only tension is applied there is a considerable out-of-plane angle near the surface because the crack orientation is now such that tension-only load is producing a local mixed mode effect. As the torsion reaches about 50% of the total value the growth direction reduces to almost zero for all nodes i.e. the load level and crack orientation are balanced such that mode I is instantaneously dominant everywhere along the crack front. As the torsion continues to increase the out of plane angle once again increases.



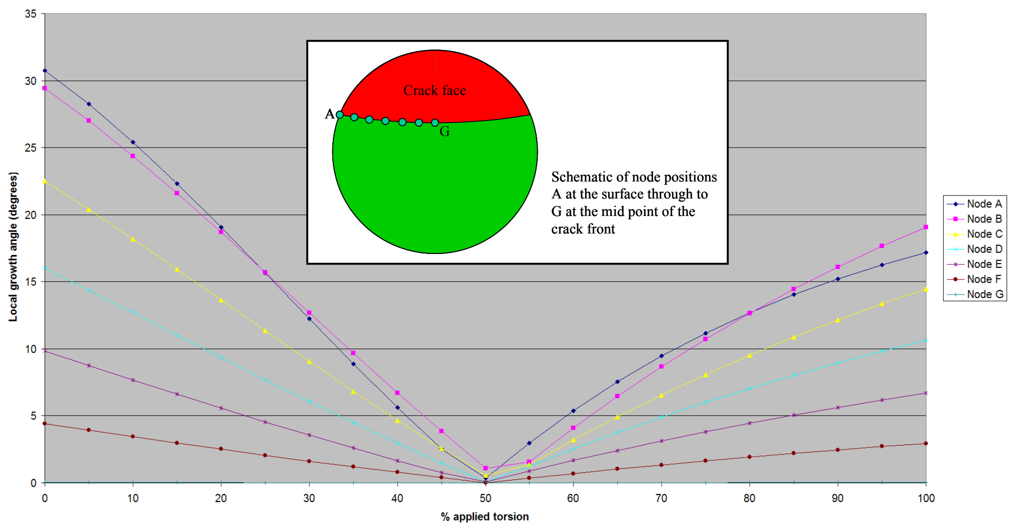


Figure 6: Constant tension and increasing torsion – instantaneous G and out-of-plane angle for nodes along an advanced crack front.

A mesh plot of the advanced crack position for this third load scenario is shown in Figure 7. Comparison of the crack trajectory shows a different overall angle of growth compared to the in-phase tension-torsion shown in Figure 4. This type of change in crack trajectory is seen in experimental tests which investigate the effect of torsion of crack growth in circular shafts e.g. [2].

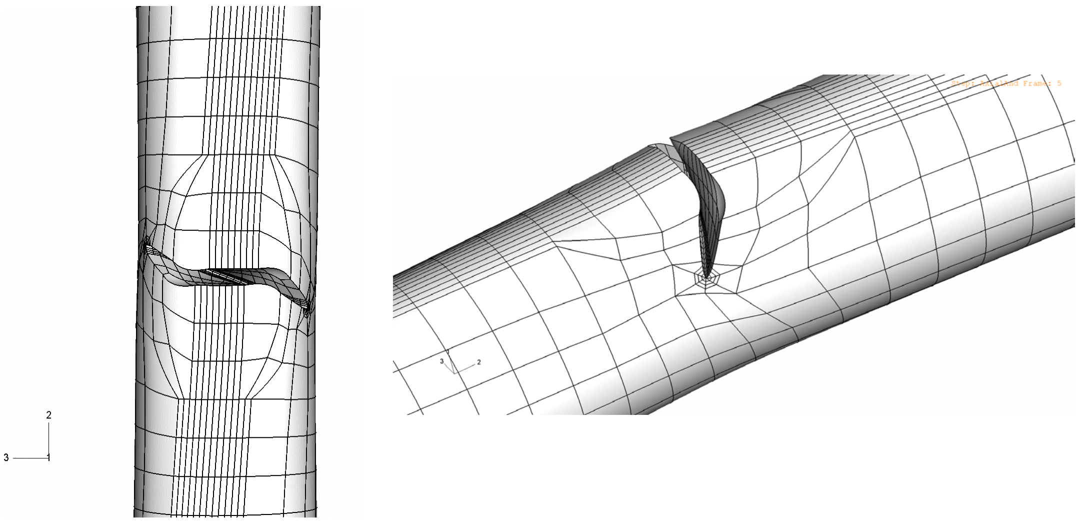
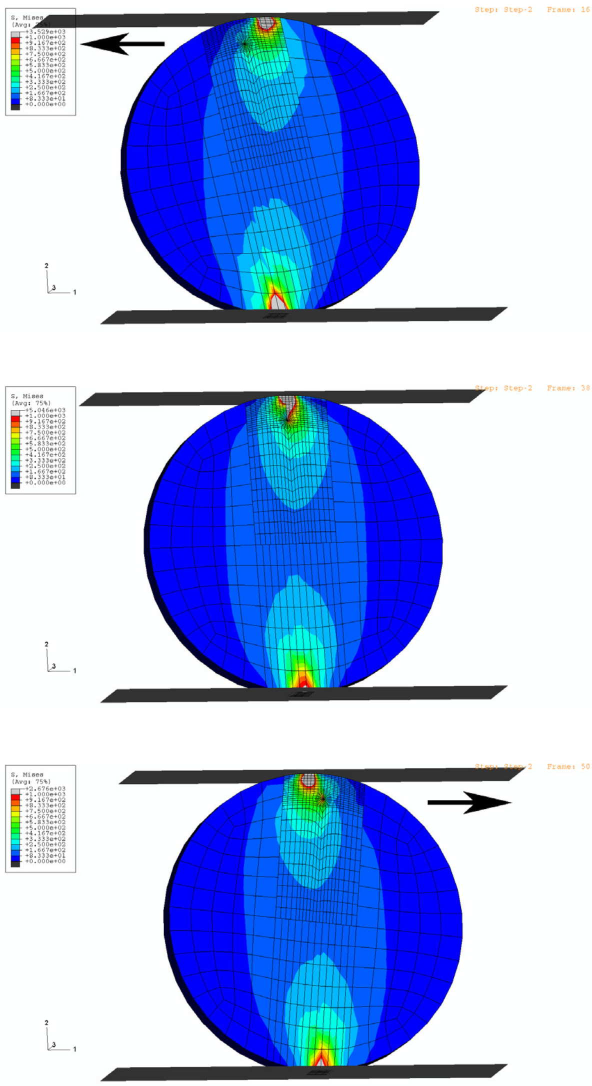


Figure 7: Stage during the constant tension-cyclic torsion loading (deformations magnified).

4: ADDITIONAL MIXED MODE EXAMPLES

Two additional examples are briefly described below. In one, the K_{II} and K_{III} effects dominate the K_I behaviour. In the other the load is non-proportional and the growth direction through the cycle needs careful treatment. (The full details cannot be shown, but these applications give an indication of the use of the simulation method for complex scenarios where the growth direction may change through the load cycle.)



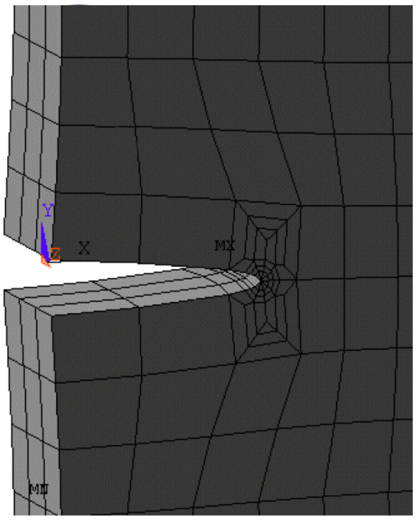
This analysis shows a simplified test model from a multi-defect assessment of a cracked roller. In the analysis the top plate compresses the roller and then slides back and forth. The plots show Mises stress for three positions during the roll cycle for a 10mm deep defect.

In this analysis the crack is expected to grow along the diameter if the roll is symmetric about the mean position. If, however, the roll extends further on one side than the other, analysis allows investigation of the effect on crack trajectory. In the full analysis the initial defects are corner cracks and mode III effects are significant.

In addition to roller supports, this type of analysis is clearly applicable to components such as rail tracks containing sub-surface defects.

Figure 8: Simplified roller support analysis.

An interesting example of thermo-mechanical analysis is contained in [3]. In this paper the techniques described here are used to assess the detailed behaviour of a specimen through a thermal load cycle whilst under a constant tension load. The aim of the analysis is to optimise a thermal loading sequence that will minimize the real time required to generate a starter crack in a specimen. Such cracks are too small to be generated in specimens using conventional mechanical cutting techniques and historical attempts to use a thermal load cycle have led to elapsed time that are unacceptably long.



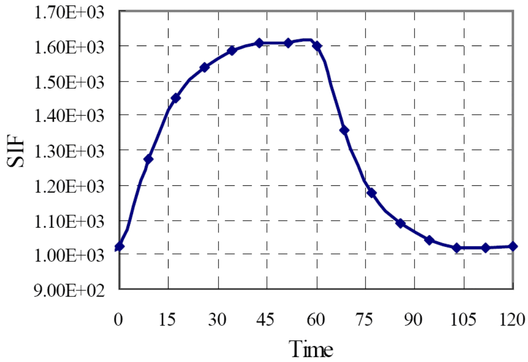


Figure 9: Stress intensity factor through a thermal cycle with constant applied tension [3].

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